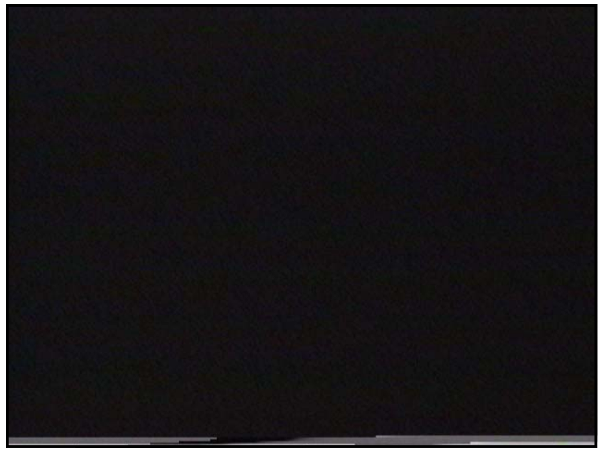
Video Segment

Juggling Robot, Dan Koditschek
University of Michigan,
ISRR'93 video proceedings

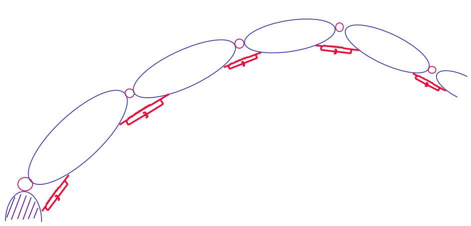


Robot Control

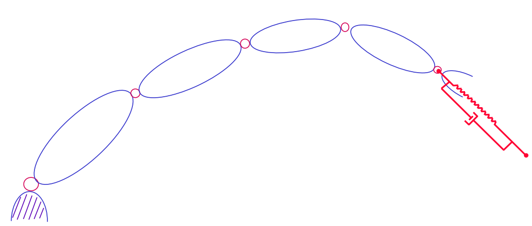
Control

- Natural Systems
- PID Control
- Joint-Space Dynamic Control
- Task-Oriented Control
- Force Control

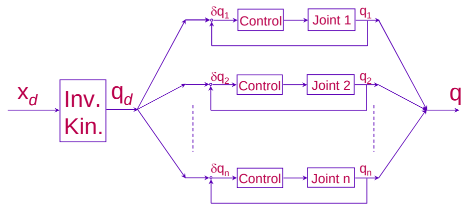
Joint-Space Control



Task-Oriented Control



Joint Space Control



Resolved Motion Rate Control (Whitney 72)

$$\delta x = J(\theta)\delta\theta$$

Outside singularities

$$\delta\theta = J^{-1}(\theta)\delta x$$

Arm at Configuration θ

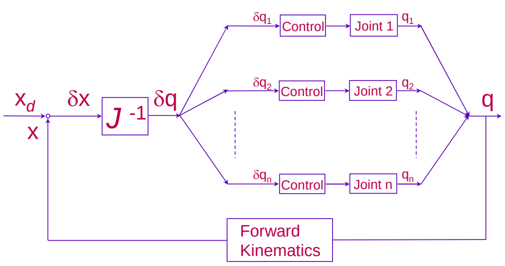
$$x = f(\theta)$$

$$\delta x = x_d - x$$

$$\delta\theta = J^{-1}\delta x$$

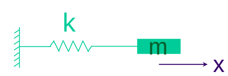
$$\boxed{\theta^{+} = \theta + \delta\theta}$$

Resolved Motion Rate Control



Natural Systems

Conservative Systems



$$\frac{d}{dt}\left(\frac{\partial(K-V)}{\partial \dot{x}}\right) - \frac{\partial(K-V)}{\partial x} = 0 \qquad K = \frac{1}{2}m\dot{x}^2$$

Natural Systems

Conservative Forces

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}}\right) - \frac{\partial K}{\partial x} = -\frac{\partial V}{\partial x}$$

$$m\ddot{x} = F = -kx$$

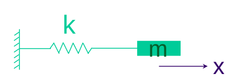
$$-\frac{\partial}{\partial x}\left(\frac{1}{2}kx^2\right)$$

Potential Energy of a spring

$$V = Work = \int_x^0 (-kx)\delta x = \frac{1}{2}kx^2$$

Natural Systems

Conservative Forces



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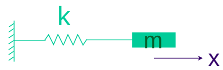
Natural Systems

Conservative Systems



$$\frac{d}{dt}\left(\frac{\partial(K-V)}{\partial\dot{x}}\right)-\frac{\partial(K-V)}{\partial x}=0$$

$$K=\frac{1}{2}m\dot{x}^2$$

$$V=\frac{1}{2}kx^2$$

$$m\ddot{x}+kx=0$$

Natural Systems

Conservative Systems

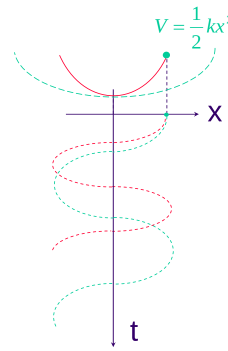
$$m\ddot{x}+kx=0$$

Frequency increases with stiffness and inverse mass

Natural Frequency $\omega_n=\sqrt{\frac{k}{m}}$

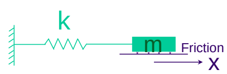
$$\ddot{x}+\omega_n^2x=0$$

$$x(t)=c\ \cos(\omega_n t+\phi)$$



Natural Systems

Dissipative Systems



$$\frac{d}{dt}\left(\frac{\partial(K-V)}{\partial\dot{x}}\right)-\frac{\partial(K-V)}{\partial x}=f_{friction}$$

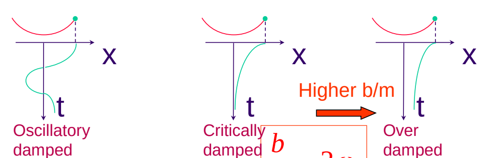
Viscous friction: $f_{friction}=-b\dot{x}$

$$m\ddot{x}+b\dot{x}+kx=0$$

Dissipative Systems

$$m\ddot{x}+b\dot{x}+kx=0$$

$$\ddot{x}+\frac{b}{m}\dot{x}+\frac{k}{m}x=0$$



$$\frac{b}{m}=2\omega_n$$

2^{d} order systems

$$m\ddot{x}+b\dot{x}+kx=0$$

$$\ddot{x}+\frac{b}{m}\dot{x}+\frac{k}{m}x=0$$

$\frac{b/m}{2\omega_n}$; $2\omega_n$; ω_n^2

Natural damping ratio

Critically damped when $b/m=2\omega_n$

$$\xi_n=\frac{b}{2\omega_n m}=\frac{b}{2\sqrt{km}}$$

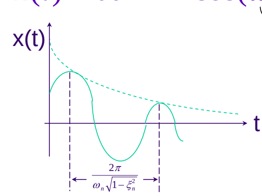
Critically damped system: $\xi_n=1\ \ (b=2\sqrt{km})$

Time Response

$$\ddot{x}+2\xi_n\omega_n\dot{x}+\omega_n^2x=0$$

Natural frequency $\omega_n=\sqrt{\frac{k}{m}}$; $\xi_n=\frac{b}{2\sqrt{km}}$ Natural damping ratio

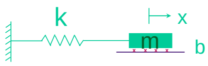
$$x(t)=ce^{-\xi_n\omega_n t}\cos(\underbrace{\omega_n\sqrt{1-\xi_n^2}}_{\omega}\,t+\phi)$$



ω damped Natural frequency

$$\omega=\omega_n\sqrt{1-\xi_n^2}$$

Example



$$m\ddot{x} + b\dot{x} + kx = 0$$

$m = 2.0$
$b = 4.8$
$k = 8.0$

what is the "damped Natural frequency"

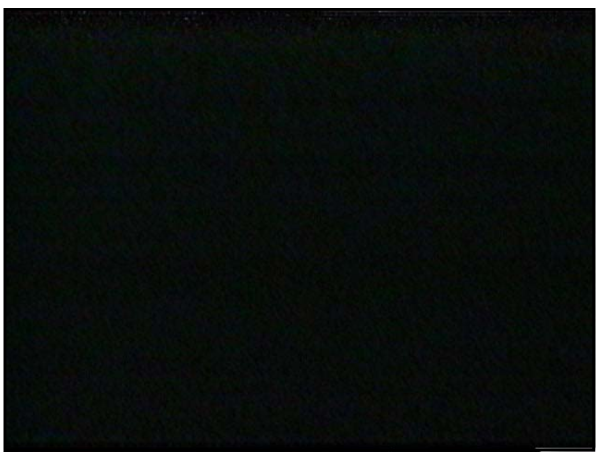
$$\omega = \omega_n \sqrt{1-\xi_n^2}$$

$$\omega_n = \sqrt{\frac{k}{m}} = 2 \; ; \quad \xi_n = \frac{b}{2\sqrt{km}} = 0.6$$

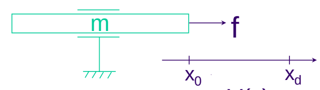
$$\omega = 2\sqrt{1-0.36} = 1.6$$

Video Segment

Tactile Sensing, H. Maekawa et al.
MEL, AIST-MITI, Tsukuba, Japan
ICRA'93 video proceedings

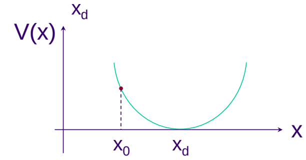


1-dof Robot Control



$$m\ddot{x} = f$$

Potential Field



$V(x) > 0, x \neq x_d$

$V(x) = 0, x = x_d$

$$V(x) = \frac{1}{2}k_p(x-x_d)^2 \; ; \; f = -\nabla V(x) = -\frac{\partial V}{\partial x}$$

$$m\ddot{x} = -\frac{\partial}{\partial x}[\frac{1}{2}k_p(x-x_d)^2] \; ; \; m\ddot{x} + k_p(x-x_d) = 0$$

Position gain

Passive Systems (Stability)

$$V_{goal} = \frac{1}{2}k_p\left(x-x_g\right)^T\left(x-x_g\right)$$

System
$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}}\right) - \frac{\partial K}{\partial x} = f$$

$$\Downarrow \quad f = -\frac{\partial V_{goal}}{\partial X}$$

Conservative Forces

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}}\right) - \frac{\partial\left(K - V_{goal}\right)}{\partial x} = 0$$

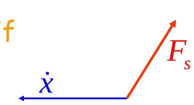
Stable

Asymptotic Stability

a system
$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}}\right) - \frac{\partial\left(K - V_{goal}\right)}{\partial x} = 0 \; F_s$$

is asymptotically stable if

$$F_s^T\dot{x} < 0 \quad ; \; for \; \dot{x} \neq 0$$



$$F_s = -k_v\dot{x} \quad \rightarrow \quad k_v > 0$$

Control

$$F = -k_p\left(x - x_{goal}\right) - k_v\dot{x}$$

Proportional-Derivative Control (PD)

$$m\ddot{x} = f = -k_p(x - x_d) - k_v\dot{x}$$

$$m\ddot{x} + k_v\dot{x} + k_p(x - x_d) = 0$$

Velocity gain (k_v) Position gain (k_p)

$$1.\ddot{x} + \frac{k_v}{m}\dot{x} + \frac{k_p}{m}(x - x_d) = 0$$

$$1.\ddot{x} + 2\xi\omega\dot{x} + \omega^2(x - x_d) = 0$$

$\xi = \dfrac{k_v}{2\sqrt{k_p m}}$ closed loop damping ratio

$\omega = \sqrt{\dfrac{k_p}{m}}$ closed loop frequency

Gains

$$k_p = m\omega^2$$

$$k_v = m(2\xi\omega)$$

Gain Selection

set $\begin{pmatrix}\xi \\ \omega\end{pmatrix} \rightarrow \begin{matrix} k_p = m\omega^2 \\ k_v = m(2\xi\omega)\end{matrix}$

Unit mass system

$$k'_p = \omega^2$$

$$k'_v = 2\xi\omega$$

m - mass system

$$k_p = m\, k'_p$$

$$k_v = m\, k'_v$$

Control Partitioning

$$m\ddot{x} = f \Longrightarrow m\ (1.\ddot{x}) = m\ f'$$

$$f = -k_v\dot{x} - k_p(x - x_d)$$

$$f = m[-k'_v\dot{x} - k'_p(x - x_d)] = m f'$$

$$m\ddot{x} = m f' \qquad f'$$

$1.\ddot{x} = f'$ unit mass system

$$1.\ddot{x} + k'_v\dot{x} + k'_p(x - x_d) = 0$$

$$2\xi\omega \qquad \omega^2$$

Non Linearities

$$m\ddot{x} + b(x,\dot{x}) = f$$

Control Partitioning

$$f = \alpha f' + \beta$$

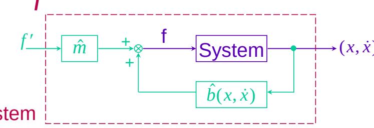
with

$$\alpha = \hat{m}$$

$$\beta = \hat{b}(x,\dot{x})$$

$$m\ddot{x} + b(x,\dot{x}) = \hat{m}f' + \hat{b}(x,\dot{x})$$

$$\longrightarrow 1.\ddot{x} = f'$$



Unit mass system

Motion Control

$$m\ddot{x} + b(x,\dot{x}) = \underset{f = mf' + b}{f} \Rightarrow 1.\ddot{x} = f'$$

Goal Position (x_d):

Control: $f' = -k'_v\dot{x} - k'_p(x - x_d)$

Closed-loop System: $1.\ddot{x} + k'_v\dot{x} + k'_p(x - x_d) = 0$

Trajectory Tracking

$$x_d(t);\ \dot{x}_d(t);\ \text{and } \ddot{x}_d(t)$$

Control: $f' = \ddot{x}_d - k'_v(\dot{x} - \dot{x}_d) - k'_p(x - x_d)$

Closed-loop System:

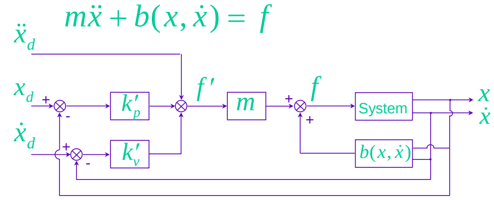
$$(\ddot{x} - \ddot{x}_d) + k'_v(\dot{x} - \dot{x}_d) + k'_p(x - x_d) = 0$$

with $e \equiv x - x_d$

$$\ddot{e} + k'_v\dot{e} + k'_p e = 0$$

Disturbance Rejection

$$m\ddot{x} + b(x,\dot{x}) = f$$



$$\ddot{e} + k'_v\dot{e} + k'_p e = 0$$

Disturbance Rejection

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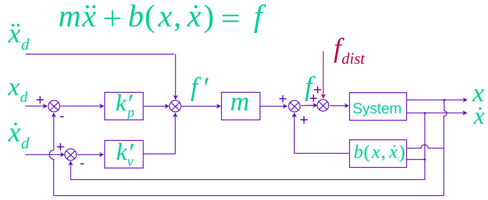


$$m\ddot{x} + b(x,\dot{x}) = f + f_{dist}$$ ← bounded $\{\forall t \,|f_{dist}| < a\}$

Control $f = mf' + b(x,\dot{x})$

Closed loop

$$\ddot{e} + k_v'\dot{e} + k_p'e = \frac{f_{dist}}{m}$$

Steady-State Error

$$\ddot{e} + k_v'\dot{e} + k_p'e = \frac{f_{dist}}{m}$$

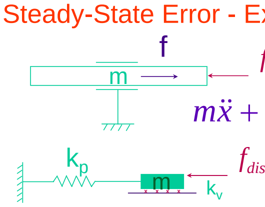
The steady-state $(\dot{e} = \ddot{e} = 0)$:

$$k_p'e = \frac{f_{dist}}{m}$$

$$e = \frac{f_{dist}}{mk_p'} = \frac{f_{dist}}{k_p}$$

← Closed loop position gain

Steady-State Error - Example



$$m\ddot{x} + k_v\dot{x} + k_p(x - x_d) = 0$$

$$k_p(x - x_d) = f_{dist}$$

$$x = x_d + \frac{f_{dist}}{k_p}$$

Δx

$$f_{dist} = k_p \Delta x$$

$$\Delta x = \frac{f_{dist}}{k_p}$$ ← Closed Loop Stiffness

PID (adding Integral action)

System $m\ddot{x} + b(x,\dot{x}) = f + f_{dist}$

Control $f = mf' + b(x,\dot{x})$

$$f' = \ddot{x}_d - k_v'(\dot{x} - \dot{x}_d) - k_p'(x - x_d) - k_i'\int(x - x_d)dt$$

Closed-loop System

$$\ddot{e} + k_v'\dot{e} + k_p'e + k_i'\int edt = \frac{f_{dist}}{m}$$

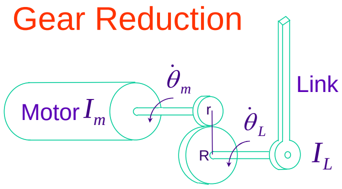
$$\dddot{e} + k_v'\ddot{e} + k_p'\dot{e} + k_i'e = 0$$

constant

Steady-state Error $e = 0$

Gear Reduction

Gear ratio $\eta = \frac{R}{r}$



$$\dot{\theta}_L = (\frac{1}{\eta})\dot{\theta}_m$$

$$\tau_L = \eta\tau_m$$

$$\tau_m = I_m\ddot{\theta}_m + \frac{1}{\eta}(I_L\ddot{\theta}_L) + b_m\dot{\theta}_m + \frac{1}{\eta}b_L\dot{\theta}_L$$

$$\ddot{\theta}_L = \frac{1}{\eta}\ddot{\theta}_m$$

$$\tau_m = (I_m + \frac{I_L}{\eta^2})\ddot{\theta}_m + (b_m + \frac{b_L}{\eta^2})\dot{\theta}_m$$

$$\tau_L = (I_L + \eta^2 I_m)\ddot{\theta}_L + (b_L + \eta^2 b_m)\dot{\theta}_L$$

Effective Inertia — Effective Damping

Effective Inertia

$$I_{eff} = I_L + \eta^2 I_m$$

for a manipulator $I_L = I_L(q)$

$\eta = 1$ Direct Drive

Gain Selection

$$k_p = (I_L + \eta^2 I_m)k_p'$$

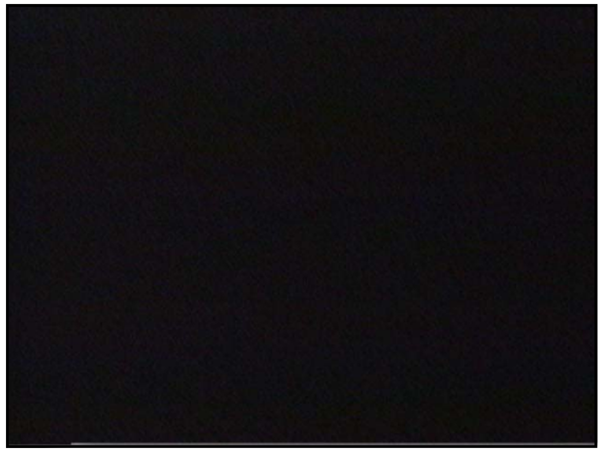
$$k_v = (I_L + \eta^2 I_m)k_v'$$

Time Optimal Selection

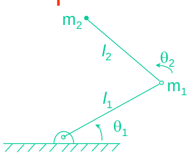
$$\hat{I}_L = \frac{1}{4}(\sqrt{I_{L_{min}}} + \sqrt{I_{L_{max}}})^2$$

Video Segment

On the Run, Marc Raibert, MIT
ISRR'93 video proceedings



Manipulator Control

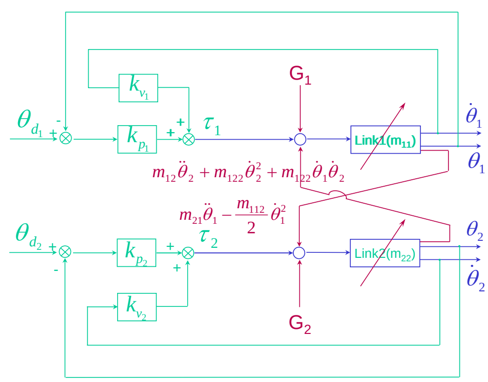


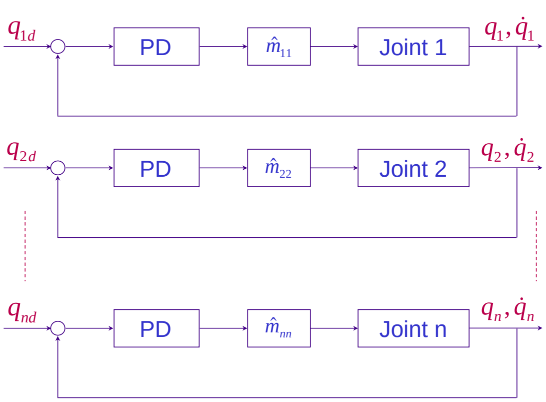
$$M(\theta)\ddot{\theta} + V(\theta,\dot{\theta}) + G(\theta) = \tau$$

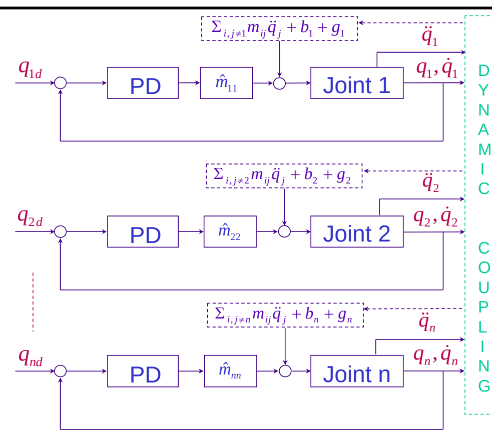
$$\begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}\begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix} + \begin{pmatrix} m_{112} \\ 0 \end{pmatrix}\begin{pmatrix} \dot{\theta}_1 & \dot{\theta}_2 \end{pmatrix} + \begin{pmatrix} 0 & m_{122} \\ -\frac{m_{112}}{2} & 0 \end{pmatrix}\begin{pmatrix} \dot{\theta}_1^2 \\ \dot{\theta}_2^2 \end{pmatrix} + \begin{pmatrix} G_1 \\ G_2 \end{pmatrix} = \begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix}$$

$$m_{11}\ddot{\theta}_1 + m_{12}\ddot{\theta}_2 + m_{112}\dot{\theta}_1\dot{\theta}_2 + m_{122}\dot{\theta}_2^2 + G_1 = \tau_1$$

$$m_{22}\ddot{\theta}_2 + m_{21}\ddot{\theta}_1 - \frac{m_{112}}{2}\dot{\theta}_1^2 + G_2 = \tau_2$$







PD Control Stability

$$M(q)\ddot{q}+B(q)[\dot{q}\dot{q}]+C(q)[\dot{q}^2]+G(\theta)=\tau$$

$$\tau=-k_p(q-q_d)-k_v\dot{q}$$

$$V_d = 1/2k_p(q-q_d)^2$$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}}\right)-\frac{\partial K}{\partial q}+\frac{\partial V_s}{\partial q}=\tau-\frac{\partial V_d}{\partial q}-k_v\dot{q}$$

PD Control Stability

$$M(q)\ddot{q}+B(q)[\dot{q}\dot{q}]+C(q)[\dot{q}^2]+G(\theta)=\tau$$

$$\tau=-k_p(q-q_d)-k_v\dot{q}$$

$$V_d = 1/2k_p(q-q_d)^T(q-q_d)$$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}}\right)-\frac{\partial K}{\partial q}+\frac{\partial (V_s-V_d)}{\partial q}=\tau_s$$

$\tau_s = -k_v\dot{q}$ with $\tau_s^T\dot{q} < 0$ *for* $\dot{q} \neq 0;\ k_v > 0$

Performance

High Gains ⟶ better disturbance rejection

Gains are limited by

- structural flexibilities
- time delays (actuator-sensing)
- sampling rate

$\omega_n \le \frac{\omega_{res}}{2}$ ⟵ lowest structural flexibility

$\omega_n \le \frac{\omega_{delay}}{3}$ ⟵ largest delay $\left(\frac{2\pi}{\tau_{delay}}\right)$

$\omega_n \le \frac{\omega_{sampling-rate}}{5}$

Nonlinear Dynamic Decoupling

$$M(\theta)\ddot{\theta}+V(\theta,\dot{\theta})+G(\theta)=\tau$$

$$\tau=\hat{M}(\theta)\tau'+\hat{V}(\theta,\dot{\theta})+\hat{G}(\theta)$$

$$1.\ddot{\theta}=(M^{-1}\hat{M})\tau'+M^{-1}[(V-\hat{V})+(G-\hat{G})]$$

with perfect estimates

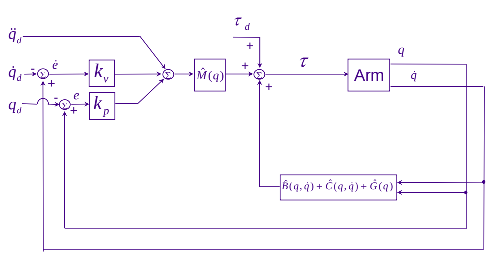
$$1.\ddot{\theta}=\tau'+\varepsilon(t)$$

τ': input of the unit-mass systems

$$\tau'=\ddot{\theta}_d-k_v'(\dot{\theta}-\dot{\theta}_d)-k_p'(\theta-\theta_d)$$

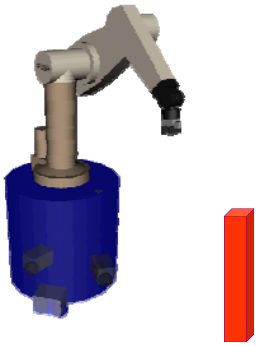
Closed-loop

$$\ddot{E}+k_v'\dot{E}+k_p'E=0+\varepsilon(t)$$

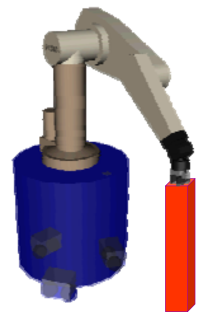


Task Oriented Control

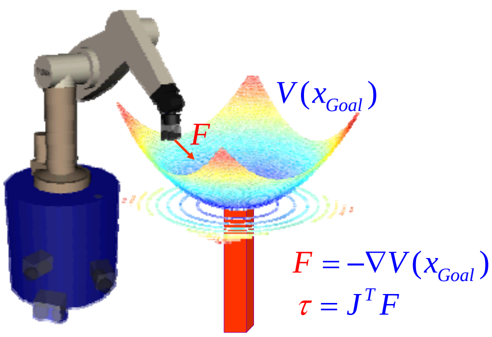
Joint Space Control



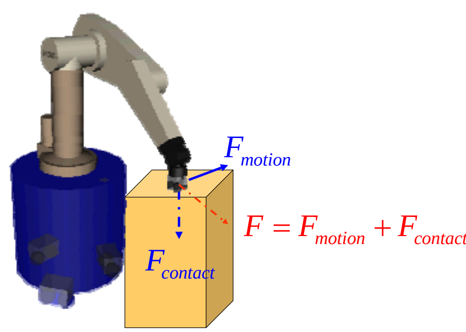
Joint Space Control



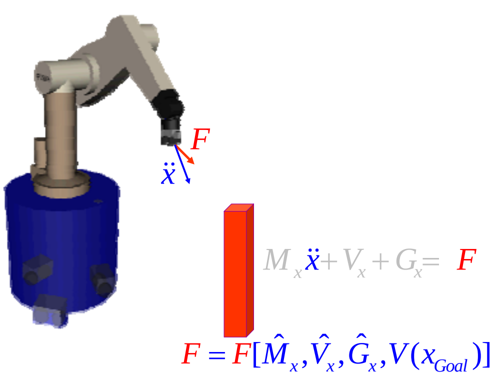
Operational Space Control



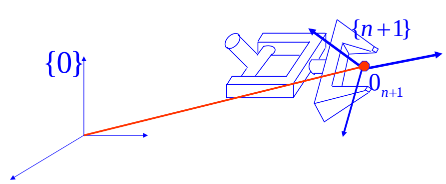
Unified Motion & Force Control



Operational Space Dynamics



Task-Oriented Equations of Motion



Non-Redundant Manipulator ; $n = m$

$$x = (x_1\, x_2 \ldots x_m)^T$$

$$q = (q_1\, q_2 \ldots q_n)^T$$

Equations of Motion

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = F \qquad x = \begin{pmatrix} x \\ y \\ z \\ \alpha \\ \beta \\ \gamma \end{pmatrix}$$

with

$$L(x,\dot{x}) = K(x,\dot{x}) - U(x)$$

Operational Space Dynamics

$$M_x(x)\ddot{x} + V_x(x,\dot{x}) + G_x(x) = F$$

x: End-Effector Position and Orientation

$M_x(x)$: End-Effector Kinetic Energy Matrix

$V_x(x,\dot{x})$: End-Effector Centrifugal and Coriolis forces

$G_x(x)$: End-Effector Gravity forces

F: End-Effector Generalized forces

Joint Space/Task Space Relationships

Kinetic Energy

$$K_x(x,\dot{x}) \equiv K_q(q,\dot{q})$$

$$\frac{1}{2}\dot{x}^T M_x(x)\dot{x} \equiv \frac{1}{2}\dot{q}^T M(q)\dot{q}$$

Using $\quad \dot{x} = J(q)\dot{q}$

$$\frac{1}{2}\dot{q}^T (J^T M_x J)\,\dot{q} \equiv \frac{1}{2}\dot{q}^T M\,\dot{q}$$

Joint Space/Task Space Relationships

$$M_x(x) = J^{-T}(q)\,M(q)\,J^{-1}(q)$$

$$V_x(x,\dot{x}) = J^{-T}(q)V(q,\dot{q}) - M_x(q)\,h(q,\dot{q})$$

$$G_x(x) = J^{-T}(q)\,G(q)$$

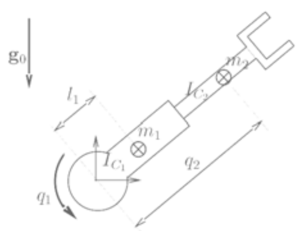
where $h(q,\dot{q}) \doteq \dot{J}(q)\dot{q}$

Example

$$q_2 = d_2$$

$$x = \begin{bmatrix} d_2 c1 \\ d_2 s1 \end{bmatrix}$$

$$^0J = \begin{bmatrix} -d_2 s1 & c1 \\ d_2 c1 & s1 \end{bmatrix}$$



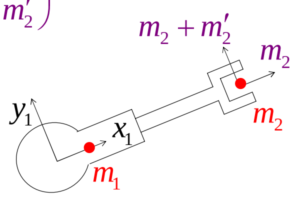
$$^0J = \begin{bmatrix} -d_2 s1 & c1 \\ d_2 c1 & s1 \end{bmatrix}$$

$$^0J = \begin{pmatrix} c1 & -s1 \\ s1 & c1 \end{pmatrix} \underbrace{\begin{pmatrix} 0 & 1 \\ d_2 & 0 \end{pmatrix}}_{^1J}$$

$$^1J^{-1} = \begin{pmatrix} 0 & 1/d_2 \\ 1 & 0 \end{pmatrix};$$

$$^1M_x = \begin{pmatrix} 0 & 1 \\ 1/d_2 & 0 \end{pmatrix} \begin{pmatrix} m_{11} & 0 \\ 0 & m_{22} \end{pmatrix} \begin{pmatrix} 0 & 1/d_2 \\ 1 & 0 \end{pmatrix}$$

$$ {}^{1}M_x = \begin{pmatrix} m_2 & 0 \\ 0 & m_2 + m_2' \end{pmatrix} $$

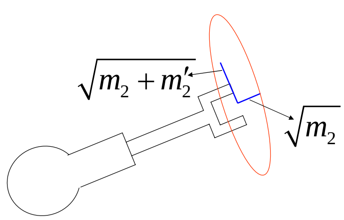


$$ m_2' = \frac{I_{331} + I_{332} + m_1 l_1^{\,2}}{d_2^{\,2}} $$

$$ {}^{0}M_x = \begin{pmatrix} c1 & -s1 \\ s1 & c1 \end{pmatrix} \begin{pmatrix} m_2 & 0 \\ 0 & m_2^{+} \end{pmatrix} \begin{pmatrix} c1 & s1 \\ -s1 & c1 \end{pmatrix} $$

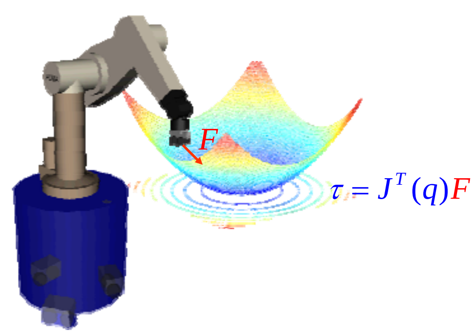
$$ m_2^{+} = m_2 + m_2' $$

$$ {}^{0}M_x = \begin{pmatrix} m_2 + m_2' s1^2 & -m_2' s\, c1 \\ -m_2' s\, c1 & m_2 + m_2' c1^2 \end{pmatrix} $$



$$ {}^{0}\Lambda = \begin{pmatrix} m_2 + m_2' s1^2 & -m_2' s\, c1 \\ -m_2' s\, c1 & m_2 + m_2' c1^2 \end{pmatrix} $$

End-Effector Control



Passive Systems (Stability)

$$ V_{goal} = \frac{1}{2} k_p \left(x - x_g\right)^T \left(x - x_g\right) $$

System $\frac{d}{dt}\left(\frac{\partial(K - V)}{\partial \dot{x}}\right) - \frac{\partial(K - V)}{\partial x} = F$

$$ \Downarrow \; F = -\frac{\partial}{\partial X}\left(V_{goal} - \hat{V}\right) $$

Conservative Forces

$$ \frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}}\right) - \frac{\partial\left(K - V_{goal}\right)}{\partial x} = 0 $$

Stable

Asymptotic Stability

a system $\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}}\right) - \frac{\partial\left(K - V_{goal}\right)}{\partial x} = 0 \; F_s$

is asymptotically stable if

$F_s^{\,T} \dot{x} < 0 \;\; ; \; for \; \dot{x} \neq 0$

$\dot{x}$ F_s

$F_s = -k_v \dot{x} \;\rightarrow\; k_v > 0$

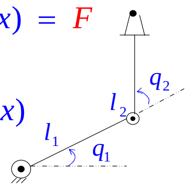
Control

$$ F = -k_p\left(x - x_{goal}\right) + \hat{G}_x - k_v \dot{x} $$

Example 2-d.o.f arm: Non-Dynamic Control

$$M_x(x)\ddot{x} + V_x(x,\dot{x}) + G_x(x) = F$$

$$F = -k_p\left(x - x_g\right) - k_v\,\dot{x} + \hat{G}(x)$$



$$\left(m_1^* c^2 12 + m_2\right)\ddot{x} + m_1^*\ddot{y} + V_{x1} = -k_p\left(x - x_g\right) - k_v\dot{x}$$

$$\left(m_1^* c^2 12 + m_2\right)\ddot{y} + m_1^*\ddot{x} + V_{x2} = -k_p\left(y - y_g\right) - k_v\dot{y}$$

Closed loop behavior

$$m_{11}(q)\ddot{x} + k_v\dot{x} + k_p\left(x - x_g\right) = -\left(m_1^*\ddot{y} + V_{x1}\right)$$

$$m_{22}(q)\ddot{y} + k_v\dot{y} + k_p\left(y - y_g\right) = -\left(m_1^*\ddot{x} + V_{x2}\right)$$

Nonlinear Dynamic Decoupling

Model

$$M_x(x)\ddot{x} + V_x(x,\dot{x}) + G_x(x) = F$$

Control Structure

$$F = \hat{M}(x)F' + \hat{V}_x(x,\dot{x}) + \hat{G}_x(x)$$

Decoupled System

$$I\,\ddot{x} = F'$$

with $\tau = J^T F$

Perfect Estimates

$$I\,\ddot{x} = F'$$

F' input of decoupled end-effector

Goal Position Control

$$F' = -k_v'\,\dot{x} - k_p'\left(x - x_g\right)$$

Closed Loop

$$I\,\ddot{x} + k_v'\dot{x} + k_p'(x - x_g) = 0$$

Trajectory Tracking

Trajectory: x_d, $\dot{x}_d$, $\ddot{x}_d$

$$F' = I\,\ddot{x}_d - k_v'(\dot{x} - \dot{x}_d) - k_p'(x - x_d)$$

$$(\ddot{x} - \ddot{x}_d) + k_v'(\dot{x} - \dot{x}_d) + k_p'(x - x_d) = 0$$

or $\ddot{\varepsilon}_x + k_v'\dot{\varepsilon}_x + k_p'\varepsilon_x = 0$

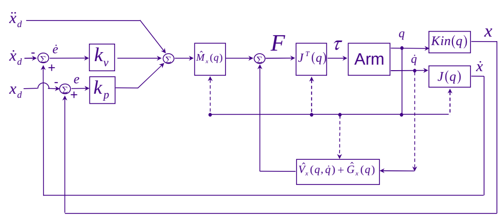
with $\varepsilon_x = x - x_d$

In joint space

$$\ddot{\varepsilon}_q + k_v'\dot{\varepsilon}_q + k_p'\varepsilon_q = 0$$

with $\varepsilon_q = q - q_d$

Task-Oriented Control

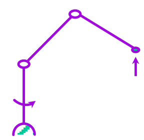


Compliance

$$I\ \ddot{x} = F'$$

$$F' = -\begin{pmatrix} k'_{p_x} & 0 & 0 \\ 0 & k'_{p_y} & 0 \\ 0 & 0 & k'_{p_z} \end{pmatrix}(x - x_d) - k'_v \dot{x}$$

set to zero



$$\ddot{x} + k'_v \dot{x} + k'_{px}(x - x_d) = 0$$

$$\ddot{y} + k'_v \dot{y} + k'_{py}(y - y_d) = 0$$

$$\ddot{z} + k'_v \dot{z} = 0$$

Compliance along Z

Stiffness

$$\ddot{z} + k'_v \dot{z} + k'_{p_z}(z - z_d) = 0$$

determines stiffness along z

Closed-Loop Stiffness: $\hat{M}_x k'_p = k_p$

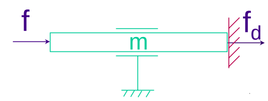
$$F = K_x (x - x_d)$$

$$\tau = J^T F = J^T K_x \Delta x = (J^T K_x J)\Delta\theta = K_\theta \Delta\theta$$

$$K_\theta = J^T(\theta) K_x J(\theta)$$

Force Control

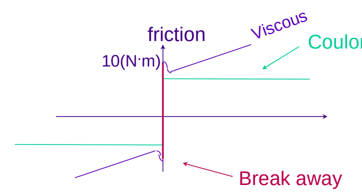
1-d.o.f.



$$m\ddot{x} = f$$

set $f = f_d$

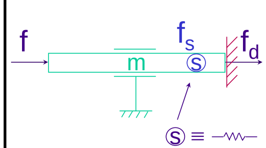
Problem



$f_d = 1Nm$

output=0

Force Sensing



$; f_s = k_s x$

$$m\ddot{x} + \underbrace{k_s x}_{f_s} = f$$

At static Equilibrium

$$f_s = f_d \Rightarrow f = f_d$$

Dynamics

$$m\ddot{x} + k_s x = f_d + f_{Dynamic}$$

Dynamics

$$m\ddot{x} + \underbrace{k_s x}_{f_s} = f$$

$$f_s = k_s x$$

$$\dot{f}_s = k_s \dot{x}$$

$$\ddot{f}_s = k_s \ddot{x}$$

$$\frac{m}{k_s}\ddot{f}_s + f_s = f$$

Control

$$f_d + \frac{m}{k_s}(-k'_{p_f}(f_s - f_d) - k'_{v_f}\dot{f}_s)$$

Closed Loop

$$\frac{m}{k_s}[\ddot{f}_s + k'_{v_f}\dot{f}_s + k'_{p_f}(f_s - f_d)] + f_s = f_d$$

Steady-State error

$$\frac{m}{k_s}(\ddot{f}_s + k'_{v_f}\dot{f}_s + k'_{p_f}(f_s - f_d)) + (f_s - f_d) = 0$$

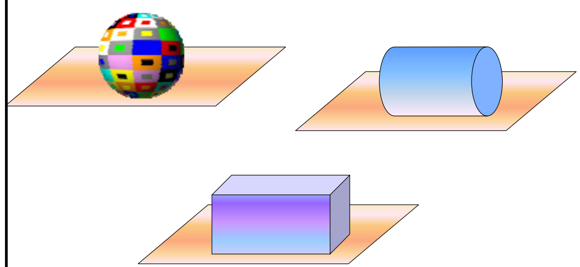
f_{dist}

$$\ddot{f}_s = \dot{f}_s = 0$$

$$\left(\frac{mk'_{p_f}}{k_s} + 1\right)e_f = f_{dist}$$

$$e_f = \frac{f_{dist}}{1 + \frac{mk'_{p_f}}{k_s}}$$

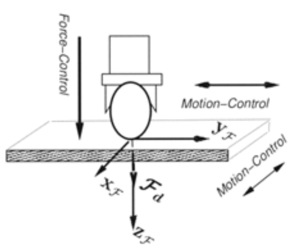
Task Description



Task Specification

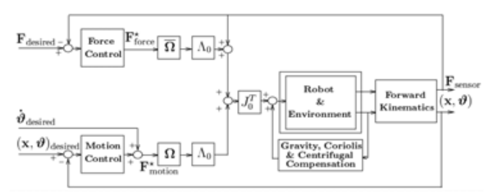
$$F = \Omega F_{motion} + \bar{\Omega} F_{force}$$

Selection matrix



$$\Omega = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad \bar{\Omega} = I - \Omega$$

Unified Motion & Force Control



Two decoupled Subsystems

$$\Omega \dot{\vartheta} = \Omega F^*_{motion}$$

$$\bar{\Omega} \dot{\vartheta} = \bar{\Omega} F^*_{force}$$