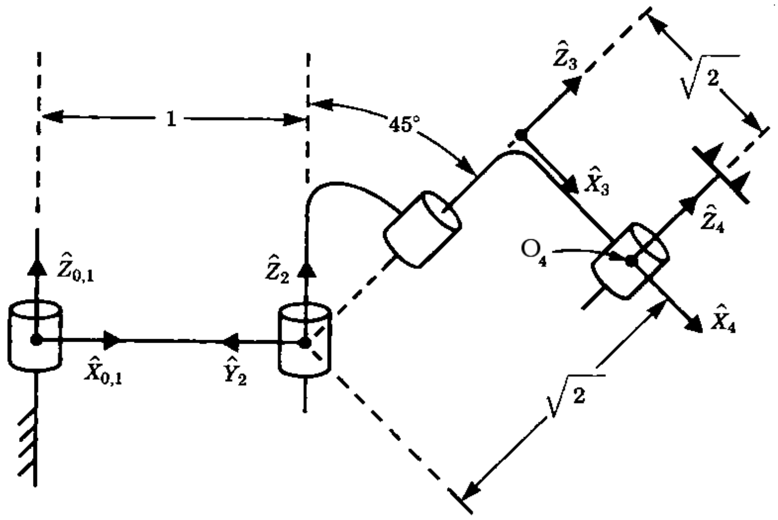
Introduction to Robotics (CS223A) Homework #4

(Winter 2007/2008) Due: **Wednesday, February 20**

1. Consider the following RRRR manipulator (image courtesy J. J. Craig):



It has the following forward kinematics and rotational Jacobian:

$$
{}^0_4T = \begin{bmatrix} c_{12}c_{34} - \frac{\sqrt{2}}{2}s_{12}s_{34} & -c_{12}s_{34} - \frac{\sqrt{2}}{2}s_{12}c_{34} & \frac{\sqrt{2}}{2}s_{12} & \sqrt{2}c_{12}c_3 - s_{12}(s_3-1) + c_1 \\ s_{12}c_{34} + \frac{\sqrt{2}}{2}c_{12}s_{34} & -s_{12}s_{34} + \frac{\sqrt{2}}{2}c_{12}c_{34} & -\frac{\sqrt{2}}{2}c_{12} & \sqrt{2}s_{12}c_3 + c_{12}(s_3-1) + s_1 \\ \frac{\sqrt{2}}{2}s_{34} & \frac{\sqrt{2}}{2}c_{34} & \frac{\sqrt{2}}{2} & s_3+1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

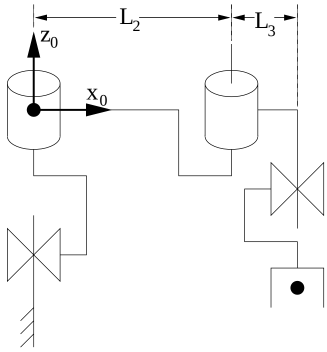
$$
{}^0J_\omega = \begin{bmatrix} 0 & 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 0 & 0 & 0 & 0 \\ 1 & 1 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}
$$

(a) Find the basic Jacobian J_o in the $\{0\}$ frame, for the position $\mathbf{q} = [0, 90^0, -90^0, 0]^T$. ($\mathbf{q}$ is the vector of joint variables.)

(b) A general force vector is applied to the origin of frame $\{4\}$ and measured in frame $\{4\}$ to be $[0, 6, 0, 7, 0, 8]^T$. For the position in (a), determine the joint torques that statically balance it.

(c) Consider the same configuration as above. A screw driver is gripped in the end-effector so that its tip is along $\hat{Z}_4$ at a distance of 9 units of length from the origin of frame $\{4\}$. What is the force and torque the screw driver tip applies when the same joint torques that were determined in part (b) are applied?

2. Consider the PRRP manipulator schematic shown below:



(a) Assuming no joint limits, sketch the workspace of this manipulator. Be sure to include dimensions in your drawing. Assume $L_2 > L_3$.

(b) Describe the (3D) dextrous workspace of this manipulator.

(c) With no joint limits, if we are considering only the position of the end effector, how many inverse kinematic solutions are there (in general)? Explain briefly.

(d) Imagine that we remove the first prismatic joint, so that the first revolute joint now rotates around the base. Repeat part (c) for such an RRP manipulator.

(e) Imagine that we further modify the manipulator from part (d) by inserting another revolute joint between the two existing revolute joints, whose axis is oriented in the same direction as the other two. Repeat part (c) for such an RRRP manipulator.

3. We wish to move a single joint from θ_0 to θ_f, starting and ending at rest, in time t_f. The values of θ_0 and θ_f are given, but we wish to calculate t_f so that these constraints hold: $|\dot{\theta}(t)| < \dot{\theta}_{max}$ and $|\ddot{\theta}(t)| < \ddot{\theta}_{max}$ for all t, where $\dot{\theta}_{max}$ and $\ddot{\theta}_{max}$ are given positive constants.

   (a) Using a single cubic segment, give equations for the cubic's coefficients a_i in terms of θ_0, θ_f and t_f.

   (b) Using the velocity constraint, $|\dot{\theta}(t)| < \dot{\theta}_{max}$, derive a condition on t_f in terms of θ_0, θ_f, and $\dot{\theta}_{max}$.

   (c) Using the acceleration constraint, $|\ddot{\theta}(t)| < \ddot{\theta}_{max}$, derive a condition on t_f in terms of θ_0, θ_f, and $\ddot{\theta}_{max}$.