1. Consider the following RRRR manipulator (image courtesy J. J. Craig):

It has the following forward kinematics and rotational Jacobian:

\[
T = \begin{bmatrix}
c_{12}c_{34} - \frac{\sqrt{2}}{2}s_{12}s_{34} & -c_{12}s_{34} - \frac{\sqrt{2}}{2} s_{12}c_{34} & \frac{\sqrt{2}}{2}s_{12} & \sqrt{2}c_{12}c_{3} - s_{12}(s_{3} - 1) + c_{1} \\
\frac{\sqrt{2}}{2}s_{12}c_{34} + \frac{\sqrt{2}}{2} c_{12}s_{34} & -s_{12}s_{34} + \frac{\sqrt{2}}{2} c_{12}c_{34} & \frac{\sqrt{2}}{2}c_{12} & \sqrt{2}s_{12}c_{3} + c_{12}(s_{3} - 1) + s_{1} \\
\frac{\sqrt{2}}{2}s_{34} & \frac{\sqrt{2}}{2}s_{34} & \frac{\sqrt{2}}{2}s_{34} & s_{3} + 1 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
J_\omega = \begin{bmatrix}0 & 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\
0 & 0 & 0 & 0 \\
1 & 1 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\end{bmatrix}
\]

(a) Find the basic Jacobian \( J_o \) in the \( \{0\} \) frame, for the position \( \mathbf{q} = [0, 90^0, -90^0, 0]^T \).

(b) A general force vector is applied to the origin of frame \( \{4\} \) and measured in frame \( \{4\} \) to be \([0, 6, 0, 7, 0, 8]^T \). For the position in (a), determine the joint torques that statically balance it.

(c) Consider the same configuration as above. A screw driver is gripped in the end-effector so that its tip is along \( \hat{Z}_4 \) at a distance of 9 units of length from the origin of frame \( \{4\} \). What is the force and torque the screw driver tip applies when the same joint torques that were determined in part (b) are applied?

2. Consider the PRRP manipulator schematic shown below:
(a) Assuming no joint limits, sketch the workspace of this manipulator. Be sure to include dimensions in your drawing. Assume $L_2 > L_3$.

(b) Describe the (3D) dextrous workspace of this manipulator.

(c) With no joint limits, if we are considering only the position of the end effector, how many inverse kinematic solutions are there (in general)? Explain briefly.

(d) Imagine that we remove the first prismatic joint, so that the first revolute joint now rotates around the base. Repeat part (c) for such an RRP manipulator.

(e) Imagine that we further modify the manipulator from part (d) by inserting another revolute joint between the two existing revolute joints, whose axis is oriented in the same direction as the other two. Repeat part (c) for such an RRRP manipulator.

3. We wish to move a single joint from $\theta_0$ to $\theta_f$, starting and ending at rest, in time $t_f$. The values of $\theta_0$ and $\theta_f$ are given, but we wish to calculate $t_f$ so that these constraints hold: $|\dot{\theta}(t)| < \dot{\theta}_{\text{max}}$ and $|\ddot{\theta}(t)| < \ddot{\theta}_{\text{max}}$ for all $t$, where $\dot{\theta}_{\text{max}}$ and $\ddot{\theta}_{\text{max}}$ are given positive constants.

(a) Using a single cubic segment, give equations for the cubic’s coefficients $a_i$ in terms of $\theta_0$, $\theta_f$ and $t_f$.

(b) Using the velocity constraint, $|\dot{\theta}(t)| < \dot{\theta}_{\text{max}}$, derive a condition on $t_f$ in terms of $\theta_0$, $\theta_f$, and $\dot{\theta}_{\text{max}}$.

(c) Using the acceleration constraint, $|\ddot{\theta}(t)| < \ddot{\theta}_{\text{max}}$, derive a condition on $t_f$ in terms of $\theta_0$, $\theta_f$, and $\ddot{\theta}_{\text{max}}$. 