Introduction to Robotics (CS223A) Homework #6

(Winter 2007/2008) Due: **Wednesday, March 12**

1. Consider the 1-DOF system described by the equation of motion, $4\ddot{x} + 20\dot{x} + 25x = f$.
   (a) Find the natural frequency ω_n and the natural damping ratio ζ_n of the natural (passive) system ($f = 0$). What type of system is this (oscillatory, overdamped, etc.) ?
   (b) Design a PD controller that achieves critical damping with a closed-loop stiffness $k_{CL} = 36$. In other words, let $f = -k_v\dot{x} - k_p x$, and determine the gains k_v and k_p. Assume that the desired position is $x_d = 0$.
   (c) Assume that the friction model changes from linear ($20\dot{x}$) to Coulomb friction, $30sign(\dot{x})$. Design a control system which uses a non-linear model-based portion with trajectory following to critically damp the system at all times and maintain a closed-loop stiffness of $k_{CL} = 36$. In other words, let $f = \alpha f' + \beta$ and $f' = \ddot{x}_d - k_v'(\dot{x} - \dot{x}_d) - k_p'(x - x_d)$. Then, find f,α,β,f',k_p' and k_v'. Note that f is an m-mass control, and f' is a unit-mass control. Use the definition of error, $e = x - x_d$.
   (d) Given a disturbance force $f_{dist} = 4$, what is the steady-state ($\ddot{e} = \dot{e} = 0$) error of the system in part (c)?

2. For a certain RR manipulator, the equations of motion are given by

$$\begin{bmatrix} 4+c_2 & 1+c_2 \\ 1+c_2 & 1 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} -s_2(\dot{\theta}_2^2 + 2\dot{\theta}_1\dot{\theta}_2) \\ s_2\dot{\theta}_1^2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

   (a) Assume that joint 2 is locked at some value θ_2 using brakes and joint 1 is controlled with a PD controller, $\tau_1 = -40\dot{\theta}_1 - 400(\theta_1 - \theta_{1d})$. What is the minimum and maximum inertia perceived at joint 1 as we vary θ_2? What are the corresponding closed-loop frequencies?
   (b) Still assuming that joint 2 is locked, at what values of θ_2 do the minimum and maximum damping ratios occur? What are the minimum and maximum damping ratios?
   (c) Now assume that both joints are free to move, and that this system is controlled by a partitioned PD controller, $\tau = \alpha\tau' + \beta$. Design a partitioned, trajectory-following controller (one that tracks a desired position, velocity and acceleration) which will provide a closed-loop frequency of 10 rad/sec on joint 1 and 20 rad/sec on joint 2 and be critically damped over the entire workspace. That is, let

$$\tau' = \ddot{\theta}_d - \begin{bmatrix} k_{v_1}' & 0 \\ 0 & k_{v_2}' \end{bmatrix} (\dot{\theta} - \dot{\theta}_d) - \begin{bmatrix} k_{p_1}' & 0 \\ 0 & k_{p_2}' \end{bmatrix} (\theta - \theta_d)$$

   then find the matrices α and β and the vector τ, along with the necessary gains k_{v_i}' and k_{p_i}'.
   (d) If $\theta_2 = 180°$, what is the steady-state error vector for a given disturbance torque, $\tau_{dist} = [2\ 4]^T$?