

Admin

- ◇ Boggle due
 - Joy poll
 - Little assign break -- use weekend to prep for midterm!
- ◇ Today's topics
 - Quicksort, generic sorting template, client callbacks
- ◇ Reading
 - Ch 7, Ch. 11.4
- ◇ Midterm next Tuesday evening
 - Terman Aud 7-9pm
- ◇ Cafe today in Terman after class

Lecture #16

Quadratic vs linearithmic

- ◇ Compare SelectionSort to MergeSort
 - 10,000 3 sec .05 sec
 - 20,000 13 sec .15 sec
 - 50,000 78 sec .38 sec
 - 100,000 5 min .81 sec
 - 200,000 20 min 1.7 sec
 - 1,000,000 8 hrs (est) 9 sec
- ◇ $O(N \log N)$ is pretty good, can we do better?
 - Theoretical result (beyond scope of 106B) no general sort algorithm better than $N \log N$
 - But a better $N \log N$ in practice?

Quicksort idea

- ◇ "Divide and conquer" algorithm
 - Divide input into low half and high half
 - Recursively sort each half
 - Join two halves together
- ◇ "Hard-split easy-join"
 - Each element examined and placed in correct half
 - Join step is trivial

Partitioning for quicksort

- ◇ Partition step uses "pivot" value
 - All elems less than pivot in one half, all elems greater in other
- ◇ How to choose pivot to get even split?
 - How to know range for values in the input at all?
- ◇ Simple choice: use first elem as pivot
 - Known to be in range at least
 - We'll examine this choice later

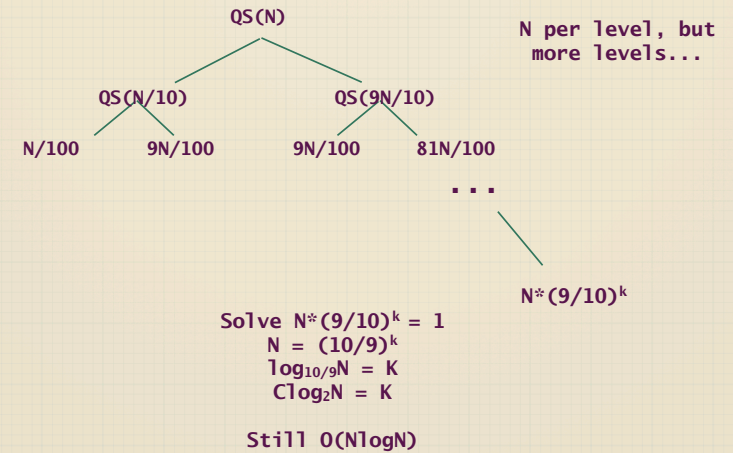
Quicksort code

```
void Quicksort(Vector<int> &v, int start, int stop)
{
    if (stop > start) {
        int pivot = Partition(v, start, stop); } O(N)
        Quicksort(v, start, pivot-1);
        Quicksort(v, pivot+1, stop);
    }
}
```

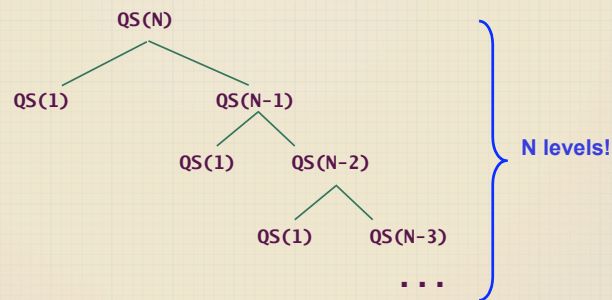
◇ Assume ideal 50/50 split

- $T(N) = N + 2T(N/2) \Rightarrow O(N \log N)$

Assuming bad 90/10 split



Assuming worst N-1/1 split



Ack! $O(N^2)$

What input has worst split?

- ◇ If pivot is smallest in input
 - Input already in sorted order!
- ◇ If pivot is largest in input
 - Input in reverse sorted order
- ◇ Others not so likely
 - Smallest, then largest, etc
- ◇ "Degenerate" case
 - May tolerate poor worst-case outcome if input is unlikely
 - Does that apply here?

What to do?

- ◇ Choose pivot differently
- ◇ Compute actual median
 - $O(N)$ algorithm exists for this
 - Guarantee 50/50 split
- ◇ "Median of three"
 - Approximate median
 - Choose middle from first, last, mid
- ◇ Random
 - Choose random element
 - Worst-case still possible, but unlikely

In clock time

- ◇ Compare MergeSort to Quicksort
 - 10,000 .05 sec .008 sec
 - 20,000 .15 sec .01 sec
 - 50,000 .38 sec .11 sec
 - 100,000 .81 sec .21 sec
 - 200,000 1.7 sec .45 sec
 - 1,000,000 9 sec 2.6 sec
- ◇ Both $O(N \log N)$ but Quicksort's advantage
 - No secondary storage
 - Moves elements more quickly to correct position

A proliferation of Swap

```
void SwapChars(char & ch1, char & ch2)
{
    char tmp = ch1;
    ch1 = ch2;
    ch2 = tmp;
}
void SwapInts(int & num1, int & num2)
{
    int tmp = num1;
    num1 = num2;
    num2 = tmp;
}
void SwapStrings(string & str1, string & str2)
{
    string tmp = str1;
    str1 = str2;
    str2 = tmp;
}
// and so on ...
```

Function template

- ◇ Same general idea as class template
 - Generic function uses same algorithm for any type
 - Write/test/debug once, use in many situations
- ◇ Template written using one or more placeholders
 - e.g. swap exchanges two values of any type
- ◇ Using function instantiates specific version
 - Call to swap passing two doubles uses a different version than a call passing two strings
- ◇ Compiler infers placeholder type if possible
 - So may not need explicit `Swap<double>`
 - (Unlike classes where `<>` always required)

Swap function template

```
template <typename Type>
void Swap(Type & one, Type & two)
{
    Type tmp = one;
    one = two;
    two = tmp;
}
```

- ◇ Template from which to create many Swap functions
 - Can create Swap for ints, chars, strings, etc. from same pattern

Using function template

```
int main()
{
    int a = 12, b = 45;
    string str1 = "apple", str2 = "orange";

    Swap(a, b);           // infers Swap<int>
    Swap(str1, str2);    // infers Swap<string>
}
```

- ◇ Compiler infers placeholder type if possible
 - Can explicitly call Swap<int>(a, b) but usually isn't necessary

Template instantiation

```
template <typename Type>
void Swap(Type & one, Type & two)
{
    Type tmp = one;
    one = two;
    two = tmp;
}
```

- ◇ What happens on call to Swap?

```
int a = 4, b = 19;
Swap(a, b);

void Swap<int>(int & one, int & two)
{
    int tmp = one;
    one = two;
    two = tmp;
}
```
- Template instantiated with Type => int
- Compiler internally names this version Swap<int>
- Code is then compiled

Instantiation errors

```
template <typename T>
void PrintVector(Vector<T> &v)
{
    for (int i = 0; i < v.size(); i++)
        cout << v[i] << " ";
    cout << endl;
}
```

← This line is trying to use << on a coordT

- ◇ Try to instantiate PrintVector for non-primitive type

```
struct coordT {
    double x, y;
};

Vector<coordT> c;
PrintVector(c);
```


Instantiation errors

◇ Compiler's response:

```
main.cpp:16: error: no match for 'operator<<' in 'std::cout
Vector<ElemType>::operator[] [with ElemType = coordT](i)'
```

◇ Template error reporting

- Template itself is largely ignored by compiler
- When called, version is created with placeholder filled in, and only then is compiled
- Errors within template now reported, triggered by client's instantiation

◇ Template may have hidden requirements on type

- e.g. Uses << to output or compares using ==
- Code instantiated won't compile if type doesn't support needed ops
- Most common operators to watch for: output, assignment, comparison/relational

Sort template

```
template <typename Type>
void Sort(Vector<Type> &v)
{
    for (int i = 0; i < v.size() - 1; i++) {
        int minIndex = i;
        for (int j = i+1; j < v.size(); j++) {
            if (v[j] < v[minIndex])
                minIndex = j;
        }
        Swap(v[i], v[minIndex]);
    }
}
```

◇ Template functions awesome for algorithms

- Searching (linear/binary), sorting (all varieties), median, mode, permute, summarize, remove duplicates, etc.

Client use of Sort template

```
int main()
{
    Vector<int> nums = ...;
    Sort(nums);

    Vector<string> strs = ...;
    Sort(strs);
}
```

◇ What must be true about the element type?

- Will every type work?
- Consider:

```
struct coordT {
    double x, y;
};
```

```
Vector<coordT> pts;
Sort(pts);
```

Fully generic sort

- ◇ Sort function template uses < to compare elements
 - This works for some types, but not all
- ◇ Division between client/implementor
 - Client knows how data is to be compared
 - Implementor is the one doing the actual comparing
- ◇ Need client/implementor cooperation
 - Client tells implementor how to appropriately compare elements
- ◇ Add function parameter
 - Client knows how to compare elements, it supplies this knowledge in the form of a function pointer
 - Callback function —implementation "calls back" to client

Sort template with callback fn

```
template <typename Type>
void Sort(Vector<Type> &v, int (cmp)(Type, Type))
{
    for (int i = 0; i < v.size() - 1; i++) {
        int minIndex = i;
        for (int j = i+1; j < v.size(); j++) {
            if (cmp(v[j], v[minIndex]) < 0)
                minIndex = j;
        }
        Swap(v[i], v[minIndex]);
    }
}
```

- ◇ Now can truly work for all types!
 - Client supplies function pointer to handle compare