Admin

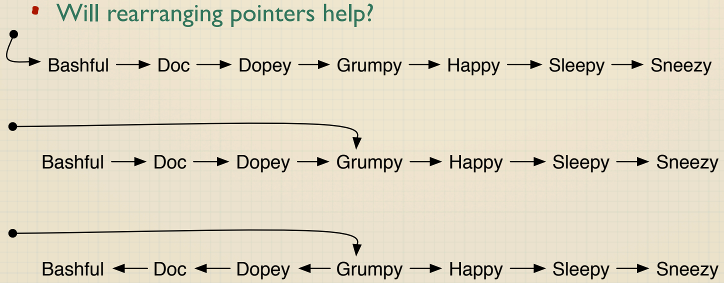
- Today's topics
  - Binary search trees, implementing Map as tree
- Reading
  - Ch 13

Map as Vector

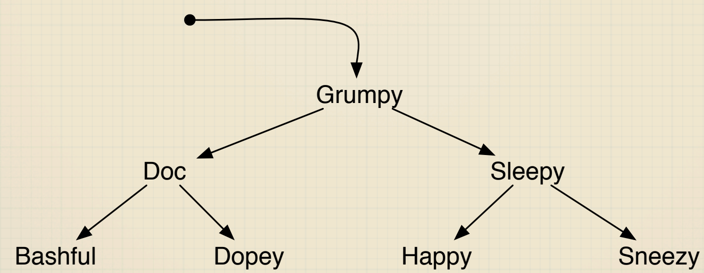
	Unsorted	Sorted
Map()	O(1)	O(1)
~Map()	O(1)	O(1)
add()	O(N)	O(N)
getValue()	O(N)	O(logN)
Overhead per entry	none	none

A different strategy

- Sorting the Vector
  - Provides fast lookup, but still slow to insert (because of shuffling)
- Does a linked list help?
  - Easy to insert, once at a position
  - But hard to find position to insert...
  - Will rearranging pointers help?



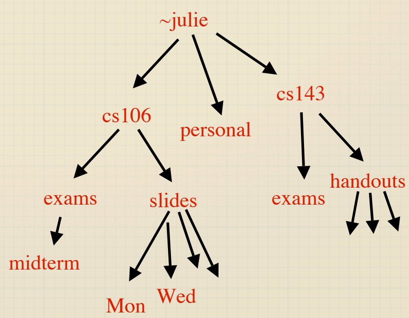
Voila... a binary search tree!



- Tree terminology
  - Node, tree, subtree, parent, child, root, leaf

Trees in general

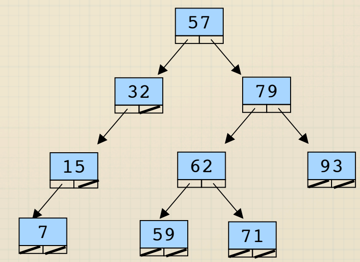
- Rules for all trees
  - Recursive branching structure
  - Single root node
  - Every node reachable from root by unique path
- Examples
  - Game tree
  - Family tree
  - Filesystem hierarchy
  - Decomposition tree
  - Binary, ternary, n-ary
  - Binary search tree



Binary search tree in specific

- Binary tree
  - Each node has at most 2 children
- Binary search tree
  - Arranged for efficient search/insert
  - All nodes in left subtree are less than root, all nodes in right subtree are greater

```
struct node {
   int val;
   node *left, *right;
};
```



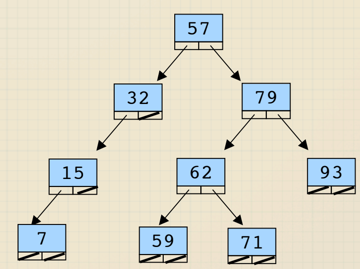
Operating on trees

- Many tree algorithms are recursive
  - Not suprisingly!
  - Handle current node, recur on subtrees
  - Base case is empty tree (NULL)
- Tree traversals to visit all nodes
  - Handle cur node, visit left/right subtrees
- Whether current node before/after its subtrees determines order of traversal
  - Pre: cur, left, right
  - In: left, cur, right
  - Post: left, right, cur
  - Others: level-by-level, reverse orders, etc.

Tree traversals at work

```
  // INORDER

void PrintTree(node *t)
{
   if (t != NULL) {
      PrintTree(t->left);
      cout << t->key << endl;
      PrintTree(t->right);
   }
}
```




```
// POSTORDER

void FreeTree(node *t)
{
   if (t != NULL) {
       FreeTree(t->left);
       FreeTree(t->right);
       delete t;
   }
}
```

Implementing Map as tree

- Each Map entry adds node to tree
  - Node contains:
    string key, client-type value, pointers to left/right subtrees
- Tree organized for binary search
  - Key is used as search field
  - Quickly find matching key or place to insert new key
- getValue
  - Searches tree, comparing keys, find existing match or error
- add
  - Searches tree, comparing keys, overwrites existing or adds new node

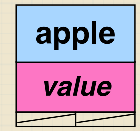
Private members for Map

```
template <typename ValType>
 class Map
 {
   public:
     // as before
  private:
    struct node {
        string key;
        ValType value;
        node *left, *right;
     };

    node *root;

    node *treeSearch(node * t, string key);
    void treeEnter(node *&t, string key, ValType val);

   DISALLOW_COPYING(Map)
};
```



Map implementation

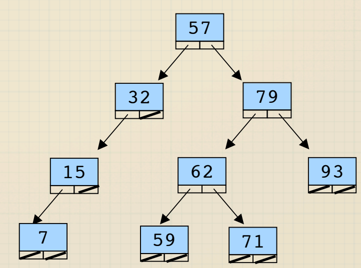
```
template <typename ValType>
  ValType Map<ValType>::getValue(string key)     // getValue is wrapper
  {
      node *found = treeSearch(root, key);       // for treeSearch rec fn
      if (found == NULL)
         Error("getValue of non-existent key!");
      else
         return found->value;
   }

template <typename ValType>
typename Map<ValType>::node *Map<ValType>::treeSearch(node *t, string key)
{
    if (t == NULL) return NULL;   // doesn't exist

    if (key == t->key)                        // found match
        return t;
    else if (key < t->key)
        return treeSearch(t->left, key);   // search left
    else
        return treeSearch(t->right, key);  // search right
}
```

Adding to a binary search tree

- Starts like getValue
  - Trace out path where node should be
- Add node as new leaf
  - Don't change any other nodes/pointers
  - Correct place to maintain binary search property



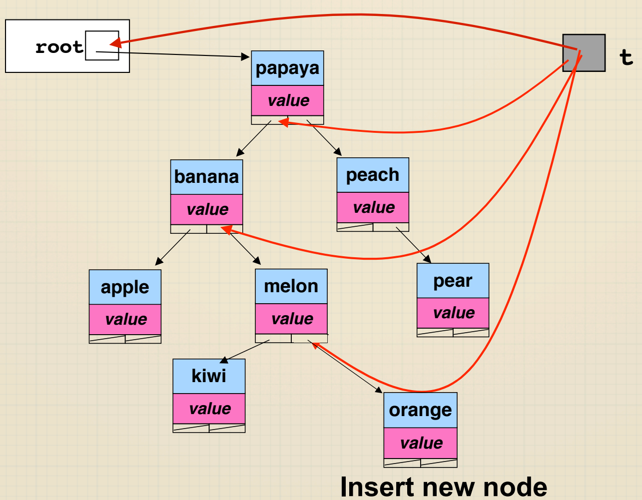
Map implementation

```
template <typename ValType>
  void Map<ValType>::add(string key,ValType val)  // add is wrapper
  {
     treeEnter(root, key, val);       // call rec helper to do enter
  }
```

Recursive treeEnter

```
template <typename ValType>
  void Map<ValType>::treeEnter(node * & t, string key, ValType val)
  {
    if (t == NULL) {
        t = new node;
        t->key = key;
        t->value = val;
        t->left = t->right = NULL;
    } else if (key == t->key) {
        t->value = val;
    } else if (key < t->key) {
        treeEnter(t->left, key, val);
    } else {
       treeEnter(t->right, key, val);
    }
 }
```

Trace treeEnter

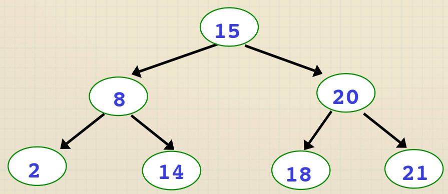


Evaluate Map as tree

- Space used
  - Overhead of two pointers per entry (typically 8 bytes total)
  - Tree adds nodes as needed, no excess capacity maintained
- Runtime performance
  - Add/getValue take time proportional to tree height
  - Height *expected* to be O(logN)

A balanced tree

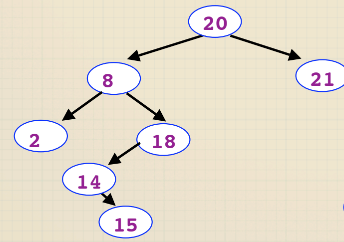
- Values: 2 8 14 15 18 20 21
- Different trees possible, depends on order inserted
- 7 nodes, expected height lg7 ≈ 3
- Perfectly balanced



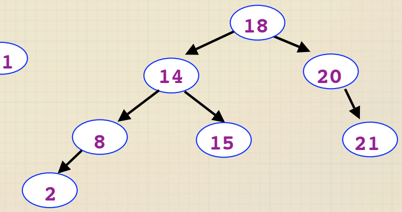
Entered: 15 8 2 20 21 14 18
(one possibility)

Mostly balanced trees

- Same values: 2 8 14 15 18 20 21
- Mostly balanced, height 4 or 5



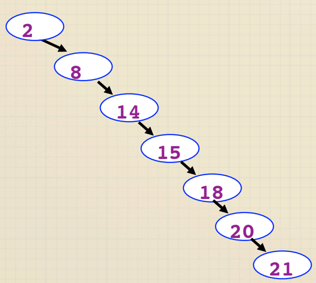
Entered: 20 8 21 18 14 15 2
(one possibility)



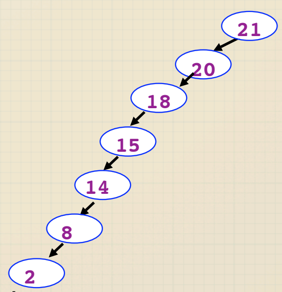
Entered: 18 14 15 8 2 20 21
(one possibility)

Degenerate trees

- Same values: 2 8 14 15 18 20 21
- Totally unbalanced, height = 7



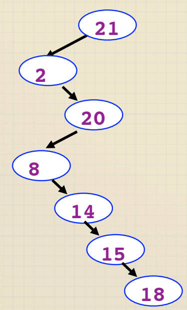
Entered: 2 8 14 15 18 20 21
(only possibility)



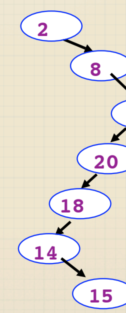
Entered: 21 20 18 15 14 8 2
(only possibility)

Even more degenerate trees

- What is relationship between worst-case inputs for tree insertion and Quicksort?



Entered: 21 2 20 8 14 15 18



Entered: 2 8 21 20 18 14 15

What to do about it?

- Might ignore degenerate outcomes if rare
  - But does that apply here?
- Wait til problem then re-balance entire tree
  - Monitor height to note when out of whack
  - Copy values to array (travel inorder to get sorted)
  - Take middle element and create new root node
  - Recursively convert left/right subarrays to subtrees
- Never let it get lopsided to begin with
  - Constantly monitor balance for each subtree
  - Rebalance subtree before going too far astray

AVL trees

- Self-balancing binary search tree
- Track balance factor for each node
  - Height of right subtree - height of left subtree
- Balance factor of 0 or 1 is ok
  - Tree is within one level of balanced
- When balance factor hits 2, restructure
- "Rotation" moves nodes from heavy to light side
  - Local rearrangement around specific node
  - When finished, node has 0 balance factor

Compare Map implementations

	Vector	Sorted Vector	BST
getValue	O(N)	O(lgN)	O(lgN)
add	O(N)	O(N)	O(lgN)

- Space used
  - Vector is just key+value, no overhead
  - BST adds 8 bytes of pointers (+ balance factor?) to each entry