EE364a Final Review Session

session outline:

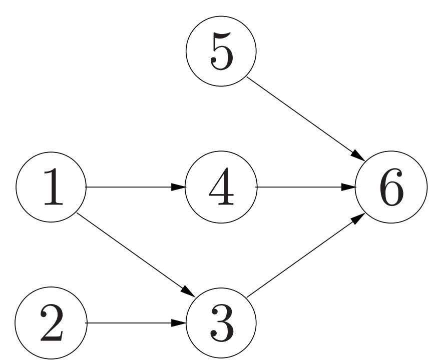
- optimizing processor speed
- $\ell_{1.5}$ optimization
- brief course overview

Optimizing processor speed

- set of n tasks to be computed by n processors
- processor power $f(s_i)$, where $s_{\min} \le s_i \le s_{\max}$ is speed
- task i completed in time $\tau_i = \alpha_i / s_i$
- total processor energy

$$E = \sum_i (\alpha_i / s_i) f(s_i)$$

- precedence constraint set $\mathcal{P} \subseteq \{1, \ldots, n\} \times \{1, \ldots, n\}$, described by graph (DAG), *e.g.*,



problem:

1. formulate the problem of minimizing completion time T subject to $E \leq E_{\max}$ as a convex optimization problem

2. generate the optimal tradeoff curve of E versus T for

$$f(s) = 1 + s + s^2 + s^3$$

issues:

- how do we deal with $E \leq E_{\max}$?

- how do we deal with precedence constraints?

Energy constraint

- energy function

$$E = \sum_i (\alpha_i / s_i) f(s_i)$$

  not convex in s in general

- write E in terms of $\tau_i = \alpha_i / s_i$

$$E = \sum_{i=1}^{n} \tau_i f(\alpha_i / \tau_i)$$

- convex (perspective)

- speed constraints become time constraints

$$\alpha_i / s_{\max} \le \tau_i \le \alpha_i / s_{\min}, \quad i = 1, \ldots, n$$

Completion time

- introduce variable t
- t_i is an upper bound on completion time of task i
- $T \leq \max_i t_i$, by construction
- precedence constraints can be expressed as

$$t_j \geq t_i + \tau_j, \quad (i,j) \in \mathcal{P}.$$

and

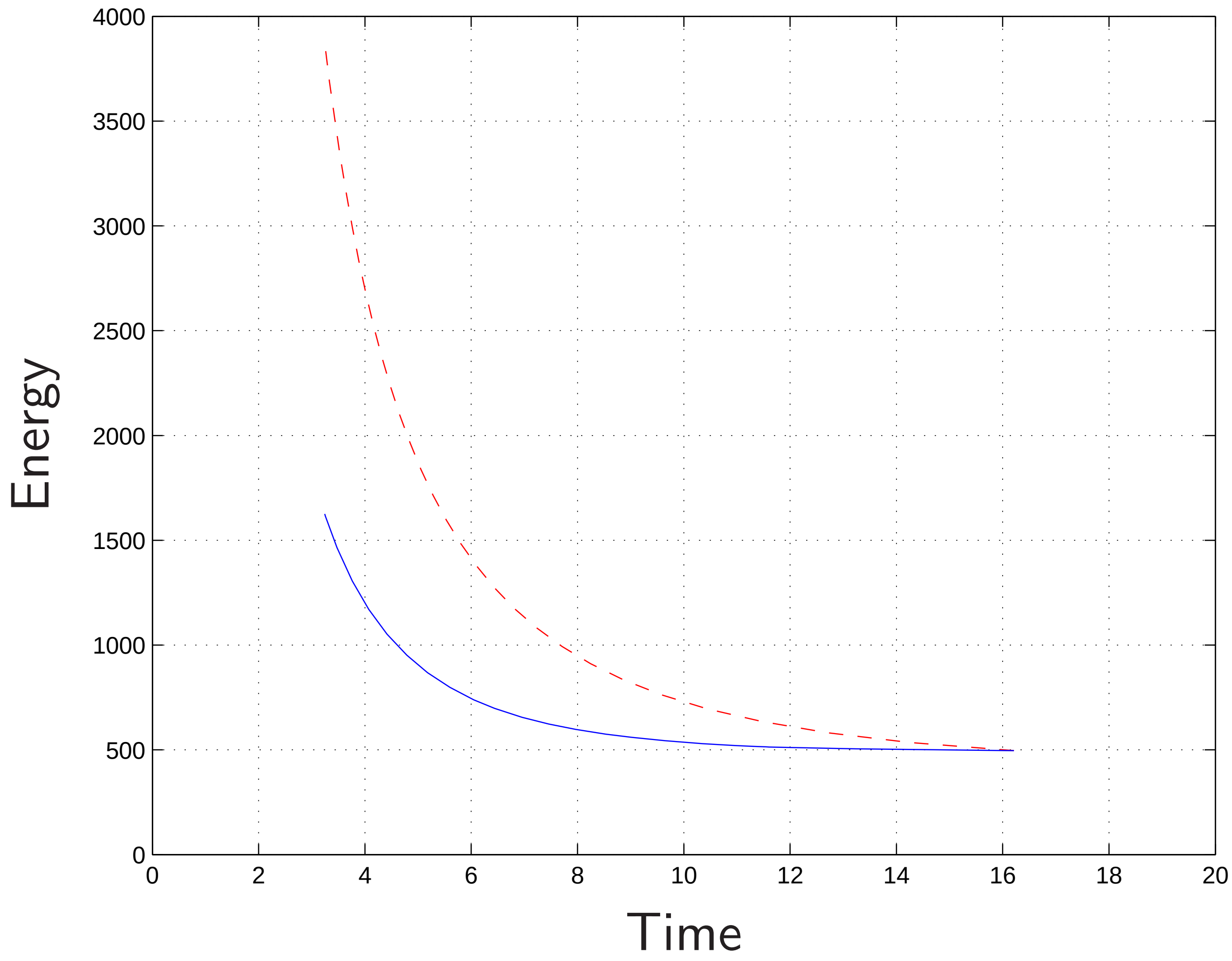
$$t_i \geq \tau_i, \quad i = 1, \ldots, n$$

Convex formulation

$$\begin{array}{ll} \text{minimize} & \max_i t_i \\ \text{subject to} & \sum_{i=1}^n \tau_i f(\alpha_i/\tau_i) \leq E_{\max} \\ & \alpha_i/s_{\max} \leq \tau_i \leq \alpha_i/s_{\min}, \quad i=1,\ldots,n \\ & t_i \geq \tau_i, \quad i=1,\ldots,n \\ & t_j \geq t_i + \tau_j, \quad (i,j) \in \mathcal{P} \end{array}$$

- variables are t and τ
- energy constraint is convex
- precedence constraints are affine
- problem is convex

Optimal tradeoff curve



$\ell_{1.5}$ optimization

$$\text{minimize} \quad \|Ax - b\|_{1.5} = \left(\sum_{i=1}^{m} |a_i^T x - b_i|^{3/2}\right)^{2/3}$$

problem:

1. give simple optimality conditions for this problem
2. formulate this problem as an SDP

Optimality conditions

equivalent problem

$$\text{minimize} \quad f(x) = \textstyle\sum_{i=1}^m |a_i^T x - b_i|^{3/2}$$

- objective differentiable
- use first order optimality conditions

$$\nabla f(x) = \sum_{i=1}^m (3/2)\,\mathbf{sgn}(a_i^T x - b_i)|a_i^T x - b_i|^{1/2} a_i = 0$$

SDP formulation

equivalent problem

$$
\begin{array}{ll}
\text{minimize} & \mathbf{1}^T t \\
\text{subject to} & s^{3/2} \preceq t, \\
 & -s_i \preceq a_i^T x - b_i \preceq s_i \quad i = 1, \ldots, m
\end{array}
$$

- variables $x \in \mathbf{R}^n$, $s, t \in \mathbf{R}^m$
- problem convex, but not an SDP
- need to transform $s^{3/2} \preceq t$ into an LMI

LMI transformation

using

$$\begin{bmatrix} u & v \\ v & w \end{bmatrix} \succeq 0 \;\Leftrightarrow\; u \geq 0,\; uw \geq v^2$$

we have that the constraint

$$s_i^{3/2} \leq t_i$$

is equivalent to

$$\begin{bmatrix} \sqrt{s}_i & s_i \\ s_i & t \end{bmatrix} \succeq 0$$

which in turn is equivalent to the LMI

$$\begin{bmatrix} y_i & s_i \\ s_i & t_i \end{bmatrix} \succeq 0, \quad \begin{bmatrix} s_i & y_i \\ y_i & 1 \end{bmatrix} \succeq 0$$

SDP formulation

putting it all together

$$
\begin{array}{ll}
\text{minimize} & \mathbf{1}^T t \\
\text{subject to} & -s_i \preceq a_i^T x - b_i \preceq s_i, \quad i = 1, \ldots, m \\
& \begin{bmatrix} y_i & s_i \\ s_i & t_i \end{bmatrix} \succeq 0, \quad \begin{bmatrix} s_i & y_i \\ y_i & 1 \end{bmatrix} \succeq 0, \quad i = 1, \ldots, m
\end{array}
$$

- SDP with variables x, s, t, and y
- same technique can be used for other problems involving polynomials
- see literature on sum of squares (SOS) methods

Brief course overview

what have you learned?

- theory
- applications
- algorithms

Theory

- convex sets and functions
- operations that preserve convexity
- convex optimization problems
- duality

Applications

- approximation and fitting
  - least-norm problems
  - robust approximation
  - function fitting
- statistical estimation
  - parametric estimation
  - optimal detector design
  - experiment design
- geometric problems
  - classification
  - placement problems
  - floor planning

many others...

Algorithms

- exploiting structure
- unconstrained minimization
  - gradient descent
  - steepest descent
  - Newton's method
- equality constrained Newton's method
- interior-point methods

Final comment

good luck on your exam!