EE364b Homework 1

1. For each of the following convex functions, explain how to calculate a subgradient at a given x.

   (a) $f(x) = \max_{i=1,\ldots,m}(a_i^T x + b_i)$.

   (b) $f(x) = \max_{i=1,\ldots,m} |a_i^T x + b_i|$.

   (c) $f(x) = \sup_{0 \leq t \leq 1} p(t)$, where $p(t) = x_1 + x_2 t + \cdots + x_n t^{n-1}$.

   (d) $f(x) = x_{[1]} + \cdots + x_{[k]}$, where $x_{[i]}$ denotes the ith largest element of the vector x.

   (e) $f(x) = \inf_{Ay \preceq b} \|x - y\|_2^2$, *i.e.*, the square of the Euclidean distance of x to the polyhedron defined by $Ay \preceq b$. You may assume that the inequalities $Ay \preceq b$ are strictly feasible.

   (f) $f(x) = \sup_{Ay \preceq b} y^T x$. (You can assume that the polyhedron defined by $Ay \preceq b$ is bounded.)

2. *A convex function that is not subdifferentiable.* Verify that the following function $f : \mathbf{R} \to \mathbf{R}$ is convex, but not subdifferentiable at $x = 0$:

$$f(x) = \begin{cases} 1 & x = 0 \\ 0 & x > 0, \end{cases}$$

   with $\mathbf{dom}\, f = \mathbf{R}_+$.