Lecture 14

Example: Aircraft dynamics

- longitudinal aircraft dynamics
- wind gust & control inputs
- linearized dynamics
- steady-state analysis
- eigenvalues & modes
- impulse matrices
...variables are (small) deviations from operating point or *trim conditions* state (components):

- $u$: velocity of aircraft along body axis
- $v$: velocity of aircraft perpendicular to body axis (down is positive)
- $\theta$: angle between body axis and horizontal (up is positive)
- $q = \dot{\theta}$: angular velocity of aircraft (pitch rate)
**Inputs**

disturbance inputs:

- $u_w$: velocity of wind along body axis
- $v_w$: velocity of wind perpendicular to body axis

control or actuator inputs:

- $\delta_e$: elevator angle ($\delta_e > 0$ is down)
- $\delta_t$: thrust

Example: Aircraft dynamics
Linearized dynamics

for 747, level flight, 40000 ft, 774 ft/sec,

\[
\begin{bmatrix}
\dot{u} \\
\dot{v} \\
\dot{q} \\
\dot{\theta}
\end{bmatrix} =
\begin{bmatrix}
-.003 & .039 & 0 & -.322 \\
-.065 & -.319 & 7.74 & 0 \\
.020 & -.101 & -.429 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
\delta_e \\
\delta_t
\end{bmatrix}
\]

• units: ft, sec, crad (\(= 0.01\) rad \(\approx 0.57^o\))

• matrix coefficients are called stability derivatives
outputs of interest:

- aircraft speed $u$ (deviation from trim)
- climb rate $\dot{h} = -v + 7.74\theta$
Steady-state analysis

DC gain from \((u_w, v_w, \delta_e, \delta_t)\) to \((u, \dot{h})\):

\[
H(0) = -CA^{-1}B + D = \begin{bmatrix}
1 & 0 & 27.2 & -15.0 \\
0 & -1 & -1.34 & 24.9
\end{bmatrix}
\]

gives steady-state change in speed & climb rate due to wind, elevator & thrust changes

solve for control variables in terms of wind velocities, desired speed & climb rate

\[
\begin{bmatrix}
\delta_e \\
\delta_t
\end{bmatrix} = \begin{bmatrix}
.0379 & .0229 \\
.0020 & .0413
\end{bmatrix} \begin{bmatrix}
u - u_w \\
\dot{h} + v_w
\end{bmatrix}
\]

Example: Aircraft dynamics
• level flight, increase in speed is obtained mostly by increasing elevator (\textit{i.e.}, downwards)

• constant speed, increase in climb rate is obtained by increasing thrust and increasing elevator (\textit{i.e.}, downwards)

(thrust on 747 gives strong pitch up torque)
Eigenvalues and modes

eigenvalues are

\[-0.3750 \pm 0.8818 j, \quad -0.0005 \pm 0.0674 j\]

• two complex modes, called \textit{short-period} and \textit{phugoid}, respectively
• system is stable (but lightly damped)
• hence step responses converge (eventually) to DC gain matrix
eigenvectors are

\[ x_{\text{short}} = \begin{bmatrix} 0.0005 \\ -0.5433 \\ -0.0899 \\ -0.0283 \end{bmatrix} \pm j \begin{bmatrix} 0.0135 \\ 0.8235 \\ -0.0677 \\ 0.1140 \end{bmatrix} , \]

\[ x_{\text{phug}} = \begin{bmatrix} -0.7510 \\ -0.0962 \\ -0.0111 \\ 0.1225 \end{bmatrix} \pm j \begin{bmatrix} 0.6130 \\ 0.0941 \\ 0.0082 \\ 0.1637 \end{bmatrix} , \]
Short-period mode

\[ y(t) = C e^{tA}(Rx_{\text{short}}) \text{ (pure short-period mode motion)} \]

- only small effect on speed \( u \)
- period \( \approx 7 \) sec, decays in \( \approx 10 \) sec

Example: Aircraft dynamics
Phugoid mode

\[ y(t) = Ce^{tA}(\mathbb{R}x_{\text{phug}}) \] (pure phugoid mode motion)

- affects both speed and climb rate
- period \( \approx 100 \) sec; decays in \( \approx 5000 \) sec

Example: Aircraft dynamics
Dynamic response to wind gusts

impulse response matrix from \((u_w, v_w)\) to \((u, \dot{h})\) (gives response to short wind bursts)

over time period \([0, 20]\):

Example: Aircraft dynamics
over time period \([0, 600]\):
Dynamic response to actuators

impulse response matrix from \((\delta_e, \delta_t)\) to \((u, \dot{h})\)

over time period \([0, 20]\):

\begin{align*}
\begin{array}{c}
\begin{array}{c}
\text{Example: Aircraft dynamics}
\end{array}
\end{array}
\end{align*}
over time period $[0, 600]$:

Example: Aircraft dynamics