EE263 homework 1 additional exercise

1. *Affine functions.* A function $f : \mathbf{R}^n \to \mathbf{R}^m$ is called *affine* if for any $x,\ y \in \mathbf{R}^n$ and any $\alpha,\ \beta \in \mathbf{R}$ with $\alpha + \beta = 1$, we have

$$f(\alpha x + \beta y) = \alpha f(x) + \beta f(y).$$

(Without the restriction $\alpha + \beta = 1$, this would be the definition of linearity.)

(a) Suppose that $A \in \mathbf{R}^{m \times n}$ and $b \in \mathbf{R}^m$. Show that the function $f(x) = Ax + b$ is affine.

(b) Now the converse: Show that any affine function f can be represented as $f(x) = Ax + b$, for some $A \in \mathbf{R}^{m \times n}$ and $b \in \mathbf{R}^m$. (This representation is unique: for a given affine function f there is only one A and one b for which $f(x) = Ax + b$ for all x.)

Hint. Show that the function $g(x) = f(x) - f(0)$ is linear.

You can think of an affine function as a linear function, plus an offset. In some contexts, affine functions are (mistakenly, or informally) called linear, even though in general they are not. (Example: $y = mx + b$ is described as 'linear' in US high schools.)