EE 261 The Fourier Transform and its Applications
Fall 2007
Course Information and Outline

Lectures: MWF 10:00 - 10:50; Skilling Auditorium
Course web site:
Please visit the web site and register.

Instructor:
Brad Osgood

Office Hours in Packard 271: Monday, Wednesday 11:00 - 12:00, Tuesday, 3:00 - 4:00, and by appointment

The course at a glance: The Fourier transform has it all. As a tool for applications it is used in virtually all areas of science and engineering. Particularly widely used is the discrete Fourier transform since computational power has increased so dramatically. In electrical engineering Fourier methods are found in all varieties of signal processing, from communications and circuit design to imaging and optics. In mathematics Fourier series and the Fourier transform are cornerstones of harmonic analysis, and the uses range from number theory to the modern formulation of the theory of partial differential equations.

Historically, the methods associated with the Fourier transform developed from employing sines and cosines to model physical phenomena that are periodic, either in time or in space. This is the subject matter of Fourier series. The desire to extend the validity of these expansions, both for an increasing array of applications and to answer purely mathematical questions, pushed the subject in several directions. On the one hand, it led to a better understanding of approximations, the use of limiting processes and the operation of integration, linear operators, and eigenfunctions and orthogonality. On the other hand, it led to the Fourier transform as a way of representing and analyzing nonperiodic phenomena.

Through the Fourier transform and its inverse we now understand that *every signal has a spectrum, and the spectrum determines the signal*, a maxim that ranks as one of the major secrets of the universe. A signal (a representation in the 'time domain') and its Fourier transform (the spectrum; a representation in the 'frequency domain') are equivalent in that one determines the other and one can pass back and forth between the two. The signal appears in different guises in the time domain and in the frequency domain however – the feel is different in the two domains – and this greatly enhances the analysis of both representations. 'Two representations for the same thing' will be an almost constant refrain in our work. In signal processing, 'filtering in the frequency domain' is an example of this, and a way of life for those who practice it. Further examples of dual representations are the sampling theorem and the Wiener-Khinchine theorem on the spectral power density. In optics, examples are diffraction and interference phenomena, in physics an example is the Heisenberg Uncertainty Principle. In mathematics, celebrated identities in number theory come from Rayleigh's identity, which, in physics, says that the energy of a signal can be computed either in time or in frequency.

The influence of Fourier analysis on mathematics itself has been profound, for in extending the applicability of the mathematical methods it became necessary to move beyond classical ideas about functions and to develop the theory and practice of distributions, also known as generalized functions. The need for such objects was recognized earlier by engineers and physicists for very practical applications.

Underlying much of this development is the notion of *linearity*, usually tagged as *linear systems* in engineering. The operations of *analyzing* a signal *into* its component parts (taking the Fourier transform) and *synthesizing* a signal *from* its component parts (taking the inverse Fourier transform) are *linear operations*. The principle of superposition applies; one can try to understand complicated signals via linear combinations of simpler signals.

We'll see some of all of this, but with such a panorama we have to be selective. The goals for the course are to gain a facility with *using* the Fourier transform, both specific techniques and general principles, and learning to recognize when, why, and how it is used. Together with a great variety the subject also has a great coherence, and my hope is that you come to appreciate both.

Topics: Here are the main topics for the course, listed more or less chronologically. Though I've separated them out to make a list, many will be mixed together and will come up in several contexts.

1. Periodicity and Fourier series in one-dimension
   - Orthogonality
   - Applications to partial differential equations
2. Definition and basic examples of Fourier transforms
   - Specific transforms
   - Shifts, scaling, modulation, differentiation
   - Fourier inversion and duality
   - Applications to diffraction
3. Convolution
   - Applications to filtering, differential equations, probability
4. Distributions (generalized functions)
   - Delta functions, generalized Fourier transforms, the *III* distribution
5. Sampling and the Nyquist theorem
   - Aliasing
6. Linear systems

- Eigenfunctions, eigenvalues, impulse response, transfer functions

7. The discrete Fourier transform and the FFT algorithm
    - Discrete convolution and digital filters
8. The two-dimensional Fourier transform
    - Two dimensional harmonics and Fourier series
    - Examples and properties of two-dimensional transforms, circularly symmetric transforms
    - Lattices and sampling
    - Crystallography
    - Line impulses, the Radon transform, the projection slice theorem
    - Medical imaging

How you should approach this class: The majority of the students in 261 are EEs, but by no means everyone, and you all bring a variety of backgrounds. Students new to the subject are introduced to a set of tools and ideas that are used by many engineers and scientists in an astonishing variety of areas. Students who have seen bits and pieces of Fourier techniques in other courses benefit from a course that puts it all together – I frequently hear this from people who have taken the class. It bears repeating that the subject has an intellectual coherence that is especially attractive to many students when they see it developed and presented with that coherence in mind. Students also benefit by seeing a greater variety of applications at a deeper level. Keep in mind the different backgrounds and interests of your fellow students, keep an open mind to the variety of the material, and let's all cut each other some slack.

Written work for the course: There will be weekly problem sets, an in-class midterm exam and a final exam. The problem sets will be assigned on Mondays and due the following Wednesday. Most problem sets will include a problem using Matlab.

Grades for the course will be based on:

- Problem sets: 35%
- Midterm: 25%
    - I would like to schedule the midterm for 90 minutes *outside of class time*, with several alternate times. I think this is much better than either a 50 minute in-class test or a take home test. Details to be announced.
- Final: 40%
    - The final exam is scheduled for Thursday, December 13 from 8:30 - 11:30 AM. There is *no* alternate time scheduled for the final.