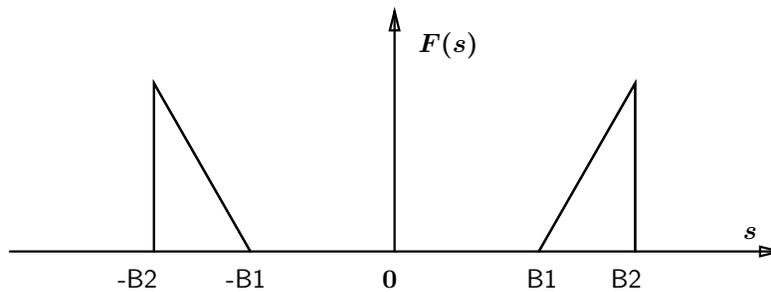


EE 261 The Fourier Transform and its Applications

Fall 2007

Problem Set 6 Due Wednesday, November 7

1. (20 points) *Nyquist rate.* The signal $f(t)$ has the Fourier transform $F(s)$ as shown below.



The Nyquist frequency is $2B_2$ since the highest frequency in the signal is B_2 . The Sampling Theorem tells us that if we sample above the Nyquist rate, no aliasing will occur. Is it possible, however, to sample at a lower frequency in this case and not get aliasing effects? If it is possible, then explain how it can be done and specify at least one range of valid sampling frequencies below the Nyquist rate that will not result in aliasing. If it is not possible, explain why not.

2. (20 points) *Natural sampling.* Suppose the signal $f(t)$ is band-limited with $\mathcal{F}f(s) = 0$ for $|s| \geq B$. Instead of sampling with a train of δ 's we sample $f(t)$ with a train of very narrow pulses. The pulse is given by a function $p(t)$, we sample at a rate T , and the sampled signal then has the form

$$g(t) = f(t) \left(\sum_{k=-\infty}^{\infty} T p(t - kT) \right)$$

- (a) Is it possible to recover the original signal f from the signal g ?
- (b) If not, why not. If it is possible, what conditions on the parameters T and B , and on the pulse $p(x)$ make it possible.
3. (15 points) Let $f(t) = \cos 2\pi t$. Suppose we sample $f(t)$ at a rate $2/3$ Hz and then interpolate using a lowpass filter with cut-off frequency $2/3$. What signal, $g(t)$, is the result? Sketch $f(t)$ and $g(t)$ on the same axes and comment on what you see. Is $g(t)$ an alias of $f(t)$ for this sampling rate?

The process illustrated in this problem is the basis of the 'sampling oscilloscope'.

4. (15 points) *More on sampling.* Let $g(t)$ be an integrable, band-limited signal with $\mathcal{F}g(s) = 0$ for $|s| \geq p/2$. Express the integral

$$\int_{-\infty}^{\infty} g(t) dt$$

in terms of sample values of g . What is the minimum sampling rate that will allow us to do this?

5. (15 points) *Matlab Exercise on Sampling:* Obtain the image <http://see.stanford.edu/materials/lsoftaee261/man.gif>

- (a) Load the 8-level gray-scale 256x256 image in Matlab using ‘imread’ command. Convert the matrix image into a vector of length 256^2 (using Matlab’s ‘:’ command) and call it the ‘time-domain’ signal $x(n)$. Change the values in $x(n)$ from type ‘uint8’ to ‘double’. Normalize it such that the maximum signal value is 1.
- (b) Find the Fourier transform $X(f)$ of $x(n)$ using the FFT command, where $X(f)$ has length the same length as $x(n)$. Plot the magnitude response of $X(f)$ centered at 0 i.e. on the X-axis, f ranges from $-\frac{256^2}{2}$ to $(\frac{256^2}{2} - 1)$ (total 256^2 points).
- (c) Define the ‘bandwidth’, $N_B(\alpha)$, of the signal $x(n)$ as the frequency values that contain fraction α of the total energy. Mathematically, this is equivalent to

$$E_{\text{total}} \triangleq \sum_{f=-\frac{256^2}{2}}^{f=\frac{256^2}{2}-1} |X(f)|^2$$

$$E_m \triangleq \sum_{f=-m}^{f=m} |X(f)|^2$$

$$N_B(\alpha) \triangleq 2p \quad \text{s.t.} \quad \frac{E_p}{E_{\text{total}}} = \alpha$$

For the following α values, $\alpha = 0.9, 0.95, 0.99$, sample the signal $x(n)$ at the corresponding Nyquist rate. In order to sample $x(n)$ at a rate $\frac{1}{r}$, simply pick every r th value in $x(n)$, so that the resulting sampled-signal $x_s(n)$ will be of length $\frac{256^2}{r}$. Find the re-constructed signal $\hat{x}(n)$ of length 256^2 using interpolation on the sampled-signal $x_s(n)$. For each value of α , the methods to be used for interpolation are (i) replication by nearest-neighbor and (ii) linear. In the two interpolation schemes, reconstruction of any point has contributions from 1 and 2 nearest neighbors respectively. Display the signal $\hat{x}(n)$ as a 256x256 image. Submit the original image, the 6 reconstructed images and Matlab code. Briefly explain the artifacts seen in the reconstructed images. Matlab commands

```
imagesc(); colormap('gray')
```

can be used to display the image.