TheFourierTransformAndItsApplications-Lecture01

Instructor (Brad Osgood):We are on the air. Okay. Welcome, one and all. And as it said on the TV when you were walking in, but just to make sure everybody knows, this is EE261, The Fourier Transform and its Applications, Fourier Transforms et al., Fourier. And my name is Brad Osgood.

Circulating around are two documents that give you information about the class. There's a general description of the class, course information, how we're gonna proceed, some basic bookkeeping items — I'll tell you a little bit more about that in just a second — and also a syllabus and a schedule, and I will also say a little bit more about that in just a second.

Let me introduce our partners in crime in this course. We have three course assistants, Thomas John — Thomas, wanna stand up? Where's Thomas? There we go. Rajiv Agarwal. Did I spell that right? Very good. Rajiv, you wanna stand up? There's Rajiv. And Nicomedus — okay so far? — M. Wanna correct that? Okay. That's Nicomedus, everybody. Thank you. All right.

And we will be setting up times for the review sessions and so on, all right? So that will be forthcoming.

We have a web page for the course. Some of you may have already visited that, but let me give you the — and the address is on one of the sheets that's being passed around, but let me write that up now so you can be sure to visit it and register for the class because it is on the web page that you will find course handouts, course information.

I will email people via the web page, all right? So you have to be registered. If I have to send an announcement to the class, post an announcement and send out an email, then that'll be done through the web page, and you have to be registered on the web page in order to get those emails. I won't be doing it through Axess, all right?

So it is at, like many of the other E classes, http:// — however you do that, wherever the colons go — where is it here? — eeclass.stanford.edu — you can find it very easily — edu/ee261, okay? Go there if you have not already and register yourself for the class. All right.

Now, let me say a little bit about the information that you have. I wanna say a little bit more about the mechanics. I'll talk more about the content in just a second. Let me say a little bit about the syllabus and schedule and the course reader. The syllabus is, as I said on the top, an outline of what we're gonna be doing, I hope a fairly accurate outline of what we're going to be doing, but it's not a contract, all right? So there will be a natural ebb and flow of the course as things go along, and when we get to particular material or what we cover in what order, this is more or less I say accurate, but it is not written in stone.

What you should use it for, however, is to plan your reading, so things will be much better for all of us if you read along with the material as the syllabus — as the schedule basically outlines, all right, because there are times when I'm gonna wanna skip around a little bit. There are times when I'm gonna derive things. There are times when I'm not gonna derive things. And you'll get much more out of the lectures, our time together, if you've read the material thoroughly before you come to class. So that's one thing I ask you to do.

We have two exams scheduled. We have a midterm exam and a final exam. I'm gonna skip to the midterm exam. Midterm exam is already actually on there, at least tentatively, sort of toward the end of October. We'll have it outside of class. That is, it'll be a sit-down, regular exam, but I wanna do it for 90 minutes rather than 50 minutes. Fifty minutes is just too short a time for material like this, so it'll be a 90-minute exam, and we'll schedule it several sessions outside of class. This is the way I've usually done it, and it hasn't been any problem. It's worked out all right for everybody. So we'll have alternate times and so on.

And the final exam is scheduled by the registrar's office. Do not come to me right before the final exam saying, "Oh, I scheduled a trip out of town. I hope that's not a problem," all right? You know what the dates are ahead of time.

We'll also have regular problem sets. None of these things that I'm saying should be new to you. You've been through the drill many times. The problem sets are gonna be — I had a startling innovation last time I taught the course where I handed out the problem sets on Monday and had them due the following Wednesday. So you actually had, like, a week and a half to do the problem sets, and there was overlap between the two. And people thought that was just a brilliant idea.

So we're gonna do that again this year except for the first problem set, all right? I decided it was not such good policy to hand out the very first problem set on the very first day of class, so I'll hand that out on Wednesday and I'll post that also — or at least I'll post it. I'm not sure I'll hand it out. It will be available on Wednesday, and it'll be due the following Wednesday. And again, these sorts of things are pretty routine for you. I'm sure you've been through them many times.

It will be practice, although again not necessarily every time without fail, to have MATLAB problems on the homework, one or two MATLAB problems on the homework. So I'm going under the assumption that people have some experience with using MATLAB — it doesn't have to be terribly advanced — and also access to using MATLAB. So if you do not have experience using MATLAB and you do not have access to MATLAB, get some experience and get some access. Won't be hard.

Okay. Now, let me say a little bit about some other things. This is the course reader for the course. It's available at the bookstore and also available on the course website, all right? It doesn't have the problems in it, but it has the material that we're gonna be covering in class.

Now, this is basically a stitched-together set of lecture notes that I've been using for a number of years in the class, and I sort of tinker with it every time I teach the class. But because it is a stitched-together set of lecture notes, the organization is sometimes a little bit odd, like you have an appendix in the middle of the chapter. And what that means is it used to be an appendix to a particular lecture that went on that particular day, and it never got moved to anywhere else, all right? So the organization can be a little bit funny.

You can help on this, all right? That is, if you find typos, if you find errors, if you find things that are less than clear in their wording, if you have some ideas for examples or other explanations, please tell me. I am working on this. I have to say that because these were written as a set of lecture notes, these are meant to be a good and I hope helpful companion for the class. That is, they're meant to be read, and they're meant to be used. So you can help, as generations of students in the past have helped, to try to refine these and turn them into something that's really a good accompaniment to the class as we go on, okay?

One other thing that's special this quarter is the class is, as always, taped, and the lectures are gonna be available to everybody. But this time for the first time, the lectures are gonna be available to the world, all right? Stanford has decided on an experimental basis — we're sort of competing with MIT here, I think — to try to make some of the materials for some classes available to the world, all right?

So the lecture notes are gonna be — everything's gonna be done through the website, but instead of needing a Stanford ID to view the taped lectures, I think anybody in the world can view these lectures. It's a little bit daunting. I have to watch my language, gotta dress well. All right. So we'll see what goes with that.

I will, however, issue a warning. I will not answer the world's email, all right? I will answer email from the class, but I will not answer — and I think I speak for the TAs here. The TAs neither will answer the world's email on this, all right? How we're gonna keep the world out of our inboxes, I'm not sure exactly whether this is gonna be a problem or not, but at any rate, that's what's happening, okay?

All right. Any questions about that? Any questions about the mechanics of the course or what your expectations should be, what my expectations of you are? Okay. All right.

Now, I always like to take an informal poll actually, when we start this class. I've taught this a number of times now, and it's always been a mixed crowd. And I think that's one of the things that's attractive about this class. So let me ask who are the EEs in this class, who are in electrical engineering, either undergraduate or graduate? All right. So that's a pretty strong show of hands. But let me also ask, who are the non-EEs in this class? All right. That's also a pretty strong show of hands.

The EEs are, as is typical, the majority of the students in the class, but there's also a pretty strong group of students in this class who are not electrical engineers by training, by desire, by anything, all right? And they usually come from all over the place. I was looking at the classes before I got to class, and I think there are some people from chemistry. Is somebody from chemistry? I thought there was somebody — yeah, I see back there. All right. And I know there were some people from earth sciences. And somebody was talking to me actually from earth sciences this morning. Somebody from earth science? Okay. Where else? I think there was an ME, couple of MEs maybe. Yeah, all right.

Now, that's important to know, I think. The course is very rich in material, all right, rich in applications, rich in content, and it appeals to many people for many different reasons, okay? For the EEs who are taking the class, you have probably seen a certain amount of this material — I don't wanna say most of the material, but you've probably seen a fair amount of this material scattered over many different classes.

But it's been my experience that one of the advantages of this class for electrical engineering students, either undergraduate or graduate students, is to see it all in one piece, all right, to put it all in your head at one time, at least once, all right, because the subject does have a great amount of coherence. It really does hang together beautifully. For all the different and varied applications, there are core ideas and core methods of the class that it is very helpful to see all at once, all right?

So if you have seen the material before, that's fine. I mean, you can draw on that and draw on your experience, but don't deny yourself the pleasure of trying to synthesize the ideas as we go along. I mean, there's nothing so pleasurable as thinking about something you already know, trying to think about it from a new perspective, trying to think about it from a new point of view, trying to fold it into some of the newer things you'll be learning.

So I have heard this from electrical engineering students many times in the past that it's a pleasure for them to see the material all together at once. It may seem like a fair amount of review, and in some cases, it will be, but not in all cases. And even if it is a review, there are often slightly different twists or slightly new takes on things that you may not have seen before or may not have thought of in quite that way. So that is my advice to the electrical engineering students.

For the students who have not seen this material before that are coming at it from a different field and maybe only heard secret tales of the Fourier transform and its uses, well, I hope you enjoy the ride because it's gonna be a hell of a ride — oh, a heck of a ride as we go along. All right.

Now, for everyone, I sort of feel like I have to issue — I don't know if I'd call this a warning or just sort of a statement or principle or whatever. This is a very mathematical class. This is one of the sort of holy trinity of classes in the information systems lab and electrical engineering. Electrical engineering is a very broad department. It's split up into a number of laboratories along research lines. I am in the information systems lab, which is sort of the mathematical part of the subject, has a lot of signal processing, coding theory, imaging, and so on.

And this course has been for a number of years taught by faculty, sort of thought of as a cornerstone in the signal processing, although it has a lot of different applications to a lot of different areas. The other courses in that holy trinity are 263, linear dynamical systems, and 278, statistical signal processing.

Actually, let me ask here because this is also very common: Who's taking 263 in the class? Also a strong majority. And who's taking 278? Yeah, okay. So there's a little bit less but still a number of people. We'll actually see, not so much with 278, well, actually with both classes, with 263 and 278, you'll actually see some overlap that I also hope you find interesting. The language will be slightly different. The perspective will be slightly different, but you see this material in this class melding over into the other classes and vice-versa. And again, I think it's something that you can really draw on and I hope you enjoy, all right?

So those classes and the perspective that we take, the faculty who are teaching this class, is a pretty mathematical one, but it's not a class in theorems and proofs. You can breathe a heavy sigh of relief now, all right? I can do that, but I won't, all right?

I will derive things — I'll derive a number of formulas — I'll derive it and I'll go through those derivations, or I'll hope that you go through the derivations in the book when I hope and I think that they will be helpful, all right? And when, in some cases, that is, there's an important technique or there's an important idea that you'll see not only in the particular instance but overall, that you'll see the same sort of derivation or the same sort of ideas be applied not only for one formula but for other sorts of formulas.

And also, in some cases, to my mind, as twisted as that may be, I sometimes think of the derivation of a formula almost as identical with the formula. I mean, to use the formula effectively almost is to know the derivation because it's to know where it applies and to know how it applies and where do we expect to use it, all right?

So that's why I will go through those things for the purpose of teaching a certain amount of technique and for the purposes of sort of having those techniques really at your fingertips so that you can apply them again in a situation that may not be quite identical with what we did but will be similar enough so that the ideas may apply in that situation. That's very important.

We will also do plenty of different sorts of applications. And again, because the field, the subject is so varied and because the clientele, because the students in the class are also varied, we'll try to take applications from different areas. We'll have applications from electrical engineering, but we'll also have applications from physics and from other areas.

I've also done in the past, and we'll see if I can get to this, some applications from earth sciences, for example. And we'll just see how that goes. So we all have to cut each other a little bit of slack, and if an application or particular area is not exactly to your liking, well, chances are it might be to somebody's liking to your right or left. So, as I say, everybody should cut each other a little slack and just enjoy the ride.

I should also say that many of the more specialized applications are found in more specialized courses, all right? So we will touch on a lot of things, and I will use the words that are used in a lot of different courses and a lot of different subjects. But we won't always do C in application to its bitter end, so to speak, or we won't do every — we certainly won't do every possible application because there are just so many of them. So you will find you will not run out of ways of using the Fourier transform and Fourier analysis techniques in any classes here. It goes on and on and on. But we'll only be able to see a certain amount of that, all right?

And actually, that leads to a very important point, at least at the start of the class; that is, where do we start, all right? That is, this subject, which is so rich and so diverse, forces you, forces me, forces all of us to make hard choices in some ways about what we're gonna cover, where we're gonna start, what direction we're going to go. And all the different choices are defensible. You will find books out there that take very different tactics towards the subject. They take different starting points; they have different emphasis; they go off in different directions. And you can make a good argument for any one of those choices. But you have to make a choice.

So for us, we are going to choose — I have chosen, not we, me — I have chosen to start the class with a brief discussion of Fourier series and go from there to the Fourier transform, all right, whereas it is also a very common choice to forget about Fourier series and maybe pick them up a little bit along the end or pick them up a little bit on the edges or, assuming that everybody's seen Fourier series, and then go right into the Fourier transform.

I don't wanna do that because I think the subject of Fourier series is interesting enough in it — we're not gonna do very much with it, but it's interesting enough in itself. Again, it's something you may have seen in different contexts, but it provides a natural transition to the study of the Fourier transform. And it is historically actually the way the subject developed, okay? So that's how we're gonna do things. We'll start with Fourier series and use them as a transition to Fourier transform.

Now, first of all, what is this concerned with overall? It may be a little bit too strong a statement, but for our purposes, I wanna identify the idea of Fourier series as almost identified with the study of periodic phenomena, all right? So for us, it's identified most strongly with a mathematical analysis of periodic phenomena.

Now, it certainly shouldn't be necessary for me to justify periodic phenomena as an important class of phenomena. You have been studying these things for your entire life pretty much. Ever since the first physics course you ever took where they do the harmonic oscillator, and then the second physics course you took where they did the harmonic oscillator, and then the third physics course you took where they did the harmonic oscillator, you have been studying periodic phenomena, all right? So that shouldn't be a controversial choice. Fourier series goes much beyond that, but it is first and foremost for us associated with the study of periodic phenomena.

The Fourier transform, although again it maybe doesn't do it justice completely, can be viewed as a limiting case of Fourier series. It has to do with a mathematical analysis on non-periodic phenomena. So if you wanna contrast Fourier series and Fourier transforms, then that's not a bad rough and ready way of doing it. As I say, it doesn't capture everything, but it captures something.

So Fourier transform as a limiting case, in a meaning that I'll make more precise later — as a limiting case of Fourier series or Fourier series techniques is identified with or has to do with — is concerned with, how about that for a weaseling way out of it? — is concerned with the analysis of non-periodic phenomena. So again, it doesn't say everything, but it says something.

And one of the things that I hope you get out of this course, especially for those of you who have had some of this material before, are these sort of broad categorizations that help you sort of organize your knowledge, all right? It's a very rich subject. You gotta organize it somehow. Otherwise, you'll get lost in the details, all right? You wanna have certain markers along the way that tell you how to think about it, how to organize it, what a particular formula, what general category it fits under, okay?

Now, it's interesting that the ideas are sometimes similar and sometimes quite different. And sometimes the situation is simpler for periodic phenomena; sometimes the situation's more complicated for periodic phenomena. So it's not as though there's sort of a one-to-one correspondence of ideas. But that's one of the things that we'll see and one of the reasons why I'm starting with Fourier series is to see how the ideas carry over from one to the other, see where they work and see where they don't work, all right? Some ideas carry easily back and forth between the two, some phenomena, some ideas, some techniques; some don't. And it's interesting to know when they do and when they don't. Sometimes the things are similar, and sometimes they're not.

Now, in both cases, there are really two kind of inverse problems. There's the question of analysis, and there's the question of synthesis, two words that you've used before, but it's worthwhile reminding you what they mean in this context. The analysis part of Fourier analysis has to do with breaking a signal or a function — I'll use the terms signal and function pretty much interchangeably, all right? I'm a mathematician by training, so I tend to think in terms of functions, but electrical engineers tend to think in terms of signals, and they mean the same thing, all right?

So analysis has to do with taking a signal or a function and breaking it up into its constituent parts, and you hope the constituent parts are simpler somehow than the complicated signal as it comes to you. So you wanna break up a signal into simpler constituent parts. I mean, if you don't talk just in terms of signals here or you don't use exactly that language, that's the meaning of the word analysis, I think, close enough; whereas, synthesis has to do with reassembling a signal or reassembling a function from its constituent parts — a signal from its constituent parts, all right?

And the two things go together, all right? You don't want one without the other. You don't wanna break something up into its constituent parts and then just let it sit there, all these little parts sitting on the table with nothing to do. You wanna be able to take those parts and maybe modify those parts, maybe see which parts are more important than other parts, and then you wanna put them back together to get either the original signal or a new signal. And the process of doing those things are the two aspects of Fourier analysis. I use the word analysis there sort of in a more generic sense.

Now, the other thing to realize about both of these procedures, analysis and synthesis, is that they are accomplished by linear operations. Series and integrals are always involved here. Both analysis and synthesis, Fourier analysis and synthesis, are accomplished by linear operations. This is one of the reasons why the subject is so — I don't know — powerful because there is such a body of knowledge, and there's such a deep and advanced understanding of linear operations, linearity.

We'll make this a little bit more explicit as we go along further, but I wanted to point it out now because I won't always point it out, all right, because when I say linear operations, what I'm thinking of here, integrals and series, all right, e.g., i.e., integrals and series, both of which are linear operations. The integral of a sum is the sum of the integrals. The integral of a constant times a function is the constant times the interval of the function, and similarly with sums, all right?

Because of this, one often says or one often thinks that Fourier analysis is part of the study of linear systems, all right? In engineering, there are courses called linear systems and so on, and sometimes Fourier analysis is thought to be a part of that because the operations involved in it are linear. I don't think of it that way. I mean, I think it's somehow important enough on its own not to think of it necessarily as subsumed in a larger subject. But nevertheless, the fact that the operations are linear does put it in a certain context; in some ways, in some cases, a more general context that it turns out to be important for many ideas, all right?

So you see — you often hear that Fourier analysis is a part of the subject of linear systems, the study of linear systems. See, I don't think that really does complete justice to Fourier analysis because of the particular special things that are involved in it, but nevertheless, you'll hear that.

Okay. Now, let's get launched, all right? Let's start with the actual subject of Fourier series and the analysis of periodic phenomenon, so periodic phenomena and Fourier series. As I said, it certainly shouldn't be necessary for me to sell the importance of periodic phenomena as something worth studying. You see it everywhere, all right?

The study of periodic phenomena is for us the mathematics and engineering or mathematics and science and engineering of regularly repeating phenomena. That's what's always involved. There's some pattern that repeats, and it repeats regularly, all right? So it's a mathematics and engineering — since this is an engineering course, I'll put that before science, or maybe I won't even mention science — mathematics and engineering of regularly repeating patterns. I'm leaving a couple of terms here — I'm leaving all these somewhat vague. What does it mean to be regular? What does it mean to be repeating? What is a pattern in the first place? But you know what I mean. You know it when you see it. And the fact that you can mathematically analyze it is what makes the subject so useful.

Now, I think, although again, it's not ironclad — the trouble is this subject is so rich that every time I make a statement, I feel like I have to qualify it. Well, it's often true, but it's not completely true, and sometimes it's not really true at all, but most of the time it's true, that it's helpful, but not always helpful, but most of the time helpful, occasionally helpful to classify periodicity as either periodicity in time or periodicity in space, all right? You often see periodic phenomena as one type or the other type, although they can overlap.

So periodic phenomena often are either periodicity in time — a pattern repeats in time over and over again. You wait long enough, and it happens again. So, for example, harmonic motion, so e.g., harmonic motion, a pendulum, a thing bobbing on a string, [inaudible] harmonic motion, or periodicity in space — I'll go over here — periodicity in space.

All right. Now, what I mean here is there is often a physical quantity that you are measuring that is living on some object in space, one dimension, two dimensions, whatever, that has a certain amount of symmetry, all right? And the periodicity of the phenomenon is a consequence of the symmetry of the object. I'll give you an example in just a second.

So here you have, say, some physical quantity, physical — not always, but often — physical quantity distributed over a region with symmetry. The region itself repeats, all right? The region itself has a repeating pattern, all right? So the periodicity of the phenomenon, the periodicity of the physical quantity that you're measuring is a consequence of the fact that it's distributed over some region that itself has some symmetry. So periodicity arises from the symmetry. Periodicity here of the physical quantity that you're measuring arises because the symmetry of the object where it's distributed, where it lives.

I'll give you an example [inaudible] from the symmetry. Matter of fact, I'll give you the example. The example that really started the subject, and we'll study this, is the distribution of heat on a circular ring, so, e.g., the distribution of heat on a circular ring, all right? So the physical quantity that you're interested in is the temperature, but it's the temperature associated with a certain region, and the region is a ring, all right? The ring has circular symmetry. It's round, okay? So you're measuring the temperature at points on the ring, and that's periodic because if you go once around, you're at the same place. So the temperature is periodic as a function of the special variable that describes where you are on the ring.

Time is not involved here. Position is involved, all right? It's periodic in space, not periodic in time, periodic in the spatial variable that gives you the position. And the periodicity arises because the object itself is symmetric because the object repeats.

This sort of example is why one often sees — and this actually turns out to be very far-reaching and quite deep — that Fourier analysis is often associated with questions of symmetry. In its sort of most mathematical form, you often find Fourier theories developed in this context and Fourier transforms developed in the context of symmetry. So you often see — so you see Fourier analysis, let me just say, Fourier analysis is often associated with problems — with analysis of questions that have some sort of underlying symmetry, so let me say often associated with problems with symmetry. I'll just leave it very general.

This is the very first problem. The problem of distribution of heat on a ring, we're gonna solve that problem. That was the problem that Fourier himself considered, all right, that introduced some of the methods into the whole subject that launched everything, all right? So again, it's not periodicity in time. It's periodicity in space.

And for those of you who have had or may have courses in this, the mathematical framework for this very general way of looking at Fourier analysis is group theory because the theory of groups in mathematics is a way of mathematizing the idea of symmetry and then one extends the ideas of Fourier analysis to take into account of groups; that is to say, to take into account the symmetry of certain problems that you're studying. And it really, as I say, it's quite far-reaching. We're not gonna do it. We'll actually have a few occasions to go into this but with a light touch, okay? I'm just giving you some indication of where the subject goes.

All right. Now, what are the mathematical descriptors of periodicity? Well, nothing I've said so far I'm sure is new to you at all. You just have to trust me that at some point before you know it, some things I say to you will be new, I hope.

But one of the mathematical descriptors of periodicity, again in the two different categories, say, the numbers, the quantities that you associate with either a phenomenon that's periodic in time or a phenomenon that's periodic in space, but periodic in time — for periodicity in time, you often use the frequency, all right? Frequency is the word that you hear most often associated with a phenomena that is periodic in time. You use frequency, the number of repetitions, the number of cycles in a second, say. If the pattern is repeating, whatever the pattern is, again if I leave that term sort of undefined or sort of vague, it's the number of repetitions of the pattern in one second or over time, all right? That's the most common mathematical descriptor of a phenomenon that's periodic in time.

For a phenomenon that's periodic in space, you actually use the period. That's the only word that's really in use in general for the particular — well, one thing at a time. So for periodicity in space, you use the period, all right? That is sort of the physical measurement of how long the pattern is before it repeats somehow, all right, the measurement of how — whether it's length or some other quantity — measurement of — let me just say how big the pattern is that repeats. They're not the same, all right? They have a different feel. They arise often from different sorts of problems.

This is probably too strong a statement, but I think it's fair to say that mathematicians tend to think in terms of mostly in periodic — they tend to think in terms of the period of a function or the period as the description of periodic behavior, whereas engineers and scientists tend to think of systems evolving in time, so they tend to think in terms of frequency. They tend to think of how often a pattern repeats over a certain period of time, all right? That's, like everything else, that statement has to be qualified, but I get tired of qualifying every statement, so I'll just leave it at that.

Now, of course, the two phenomena are not completely separate or not always completely separate. They come together. Periodicity in time and periodicity in space come together in, for example, wave motion, all right? That is a traveling disturbance, a traveling periodic disturbance. So the two notions of periodicity come together. Two notions here, periodicity in time, periodicity in space, come together in, e.g., wave motion, understood very generally here as a periodic — as a regularly repeating pattern that changes in time, that moves.

This board jumps up a little bit. I think I'd better skip it. So a regularly moving disturbance, like a group of freshmen through the quad, just they're everywhere, mostly regularly, mostly moving, all right?

Now, there again, the two descriptors come in, the frequency and the wavelength. So again, you have frequency and wavelength in a frequency nu and wavelength usually denoted by — this is for periodicity in space and then for periodicity in time frequency nu for periodicity in time. That's the number of times that it repeats in one second. This is cycles per second, the number of times that the pattern repeats in one second.

So, for example, you fix your position in space. Both time and space are involved. So you fix yourself at a point in space, and the phenomenon washing over you like a water wave, all right? And you count the number of times you're hit by a wave in a second, and that's the frequency. That's the number of times that the phenomenon comes to you. For periodicity in time, the phenomenon comes to you. For periodicity in space, you come to the phenomenon, so to speak, all right?

So I've fixed myself at a point in time. A wave washes over me at a certain characteristic frequency. Over and over again, regularly repeating, it comes to me nu times per second. The wavelength, you fix the time and allow — and see what the phenomena looks like distributed over space. So for periodicity in space, fix the time and see how the phenomena is distributed. See the pattern distributed over space, distributed — my writing is getting worse — distributed. Then the length of one of those complete [inaudible] is the period or the wavelength. Wavelength is a term that's associated with a periodicity in space for a traveling phenomena, for wave motion. So the length of the disturbance, say one complete disturbance, if I can say that, one complete pattern is the wavelength.

Now, like I say, ever since you were a kid, you've studied these things, and it's really denoted by Landen. But I bring it up here because of the one important relationship between frequency and wavelength, which we are going to see in a myriad of forms throughout the quarter. That is, in the case of wave motion, there is a relationship between the frequency and the wavelength determined by the velocity, and that can be two different phenomenon, all right?

Periodicity in time and periodicity in space may not have anything to do with each other, but if you have a wave traveling, if you have a regularly repeating pattern over time, then they do have something to do with each other and they're governed by the formula, distance equals rate times time, which is the only formula that governs motion, all right?

So the relationship between frequency and wavelength; that is, distance equals rate times time — I love writing this in a graduate course because it's the only equation in calculus actually. In all of calculus, I think this is pretty much the only equation, used in very clever ways, but the only equation.

And in our case, if the rate is the velocity of the wave, then this translates, if V is the velocity, the rate of the wave of the motion, then the equation becomes, as I'm sure you know many times, Lambda. That's the distance that the wave travels in one cycle. It's traveling at a speed, V. If it goes nu cycles in one second, then it goes one cycle in one over nu seconds.

Let me say that again to make sure I got that right. If it goes nu cycles in one second — if it rushes past you nu times in one second, then in one over nu seconds, it rushes past you once. Rushing past you once means you've got through one wavelength. So distance equals rate times time. The time it takes to go one wavelength is one over nu seconds. So I have Lambda equals V times one over nu, or Lambda nu equals V, again a formula you have seen many times.

Now, why did I say this if you've seen it many times? Because I never have the confidence that I can talk my way through that formula, for one thing, so I always have to do it. Secondly, it exhibits a reciprocal relationship between the two quantities, all right? There's a reciprocal relationship — you can see it more clearly over here where the constant of proportionality or inverse proportionality is the velocity, all right? Lambda is proportional to the reciprocal of the frequency, or the frequency is proportional to the reciprocal of the wavelength, at any rate, or expressed this way: Lamba times nu is equal to V, so there's a reciprocal relationship between the frequency and the wavelength, all right? This is the first instance when you talk about periodicity of such reciprocal relationships.

We are gonna see this everywhere, all right? It's one of the characteristics of the subject, hard to state as a general principle, but there, plain to see that in the analysis and the synthesis of signals using methods from Fourier series or Fourier analysis, there will be a reciprocal relationship between the two, between the quantities involved, all right? I'm sorry for being so general, but you'll see this play out in case after case after case, and it is something you should be attuned to, all right? All right.

So you may never have thought about this in these terms. It's a simple enough formula that you've used millions of times, all right? You may not have thought about it somehow in those terms, but I'm asking you to think about stuff you once saw in very simple contexts and how those simple ideas sort of cast a shadow into much more involved situations, all right?

The reciprocal relationship between, as we'll learn to call it, the reciprocal relationship between the two domains of Fourier analysis, the time domain and the frequency domain, or the time domain and the spatial domain, or the spatial domain and the frequency domain, and so on, is something that we will see constantly, all right? And I will point that out, but if I don't point it out, you should point it out to yourself, all right? You should be attuned to it because you will see it.

And it's one of those things that helps you organize your understanding of the material because sometimes when you're called upon to apply these ideas in some context that you haven't quite seen. You have to ask yourself — at least a good starting place is to ask yourself questions like, "Well, should I expect a reciprocal relationship here?" It might lead you to guess what the formula should be or guess what the relationship should be, so you'll say, "Well, somehow I wanna use Fourier analysis to do this problem, so I should be looking for some sort of reciprocal relationship. The quantities that I'm interested in somehow should be related in some kind of reciprocal way." And what that might mean might be more or less involved depending on the particular kind of problem, but you'll see it. Trust me, you'll see it. Okay. All right. Now, we're almost done for today.

Why does mathematics come into this in the first place? I mean, periodicity is evidently sort of a very physical-type property. Why does it allow any kind of mathematical description? Well, it does because there are very simple — maybe not too simple — mathematical functions that exhibit periodic behavior and so can be used to model periodic phenomenon. So math comes in because there are simple mathematical functions that model — that are periodic that repeat and so can be used to model periodic phenomenon.

I am speaking of course of our friends, the sine and cosine. Now, you may think, again, we've only talked about elementary things in a very elementary context. But, you know, I have a PhD in this subject, and I get excited talking about sines and cosines. I mean, and it's not just creeping old age. I mean, I think there's a lot to reflect on here and sometimes the miraculous nature of these things.

Cosine of — I'll use T as the variable. Cosine of T and sine of T are periodic of period 2 pi. That is, cosine of T plus 2 pi is equal to cosine of T for all [inaudible] of T and sine of 2 pi plus T plus 2 pi is equal to sine of T. Why? Dead silence. Because the sine and cosine are — don't tell me. I wanna do it. Because the — I'll do it over here — because the sine and the cosine are associated with periodicity in space because the sine and the cosine are associated with an object that regularly repeats. The simplest object that regularly repeats, the circle.

You didn't meet sine and cosine that way first. You met sine and cosine in terms of ratios of size, lengths of size in triangles. That's fine, but that's an incomplete definition. The real way of — not the real way, but the more sophisticated way, the ultimately more far-reaching way of understanding sine and cosine is as associated with the unit circle where a cosine of T is the X coordinate, and the sine of T is the Y coordinate, and T is a radian measure.

I'm not gonna go through this in too much detail, but the point is that the sine and the cosine are each associated with the phenomenon of periodicity in space. They are periodic because if you go once around the circle; that is to say, T goes from T to T plus 2 pi, you're back where you started from, all right? That's why. It's periodicity in space, all right?

That's the definition of sine and cosine that exhibits their periodic phenomenon, not the definition in terms of right triangles. It's not that the definition in terms of right triangles is wrong; it just doesn't go far enough. It's incomplete, all right? It doesn't reveal that fundamental link between the trigonometric functions and periodicity, and it is fundamental. If not for that, mathematics could not be brought to bear on the study of periodic phenomena.

And furthermore, it's clear — and we'll quit in just a second — that it's not just 2 pi but any multiple of 2 pi, positive or negative. I can go clockwise or I can go counter-clockwise. I can say the cosine of T plus 2 pi N is the same thing a cosine of T. And the sine of 2 pi — T plus 2 pi N is the sine of T for N — any integer — and 0 plus or minus 1, plus or minus 2, and so on and so on.

The interpretation is that when N is positive, I'm going counter — and it is just an interpretation; it is just a convention — when N is positive, I'm going counter-clockwise around the circle; when N is negative, I'm going clockwise around the circle. But it's only when you make the connection between periodicity and space and the sine and the cosine that you see this fundamental property, all right?

Now, all right, I think we made it out of junior high today. That was my goal, all right? What is most amazing and what we'll see next time is that such simple functions can be used to model the most complex periodic behavior, all right? From such simple things — from simple acorns, mighty oaks grow, or whatever bullshit — oh, excuse me — whatever stuff you learn out there — that these simple functions that are associated with such a simple phenomena can be used to model the most complex, really, the most complex periodic phenomena. And that is the fundamental discovery of Fourier series, all right? And it's the basis of Fourier analysis, and we will pick that up next time. Thank you very much. See you then.

[End of Audio]

Duration: 52 minutes