TheFourierTransformAndItsApplications-Lecture29

Instructor (Brad Osgood):Does somebody know – don't you occasionally get bad reactions to flu shots like about a week later? I got a flu shot last week, and I sort of had all these flu symptoms yesterday, like I thought I was gonna die for example. So anyway, I won't be my usual perky self today, and of course, only half the class is here, so I guess perkiness is in short supply these days. All right. I wish we had one additional day. Well first of all, let me call your attention one more time to the information about the final exam. It's next Thursday. That's a week from tomorrow from 8:30 to 11:30 in Dinkelspiel Auditorium. I'm sure you can find that. It is open books, open notes, and I'll supply the blue books, and I'll bring copies of the formula sheets, and so on, and so on. Does anybody have any questions about the exam, about the mechanics of it or anything like that? No? Everybody brimming with confidence, or about shocked and doesn't really wanna think about it right now? Okay. Like I say, I wish we had one additional day in this quarter because I [inaudible] don't quite have enough time to do what I wanna do in full detail, or even if not in full detail, just enough detail so you can see some of the finer points of the arguments. But nevertheless, I still think it's important to try to go over the main highlights of two more topics: 1.) a continuation of what we talked about last time, the general stretch theorem and actually, sort of an application and a physical manifestation of the general stretch; and then the last topic which I'll begin today and then finish up next time on medical imaging and application of the two-dimensional Fourier transform to medical imaging.

And again, I'm sorry. I really do regret not having one more day somehow that we could do this in a little bit more detail. But to give you a little – at least a sense of some of the directions that some of this material goes. So today I wanna talk about – and actually, this hearkens back both to yesterday, last time, and stuff we did earlier when we first started talking about the Shah function. I wanna talk about Shahs, lattices, and crystallography. Now I'm doing this because I really think this is a really interesting application. It's very pretty, and it just shows you somehow how some of these ideas come into play in unexpected ways. This is an application – although again unfortunately, we can't see all of the details. I just don't time to give all the details, although it's discussed in the notes. This is an application and say physical manifestation of this idea that in higher dimensions in reciprocal means inverse transpose. That's what we talked about when we talked about the general stretch theorem, and I'll remind you what that says. It's a physical manifestation – physical example. Manifestation is too long a word for a man with a grip – physical example of the idea is reciprocal means inverse transpose. We saw that phrase, that aphorism in the context of the generalized stretch theorem, so I'll remind you what that says. And that comes into the derivation of what I'm going to be doing. So we saw this in the generalized stretch theorem which told us how to take the Fourier transform one and make it change a variable by a matrix, and not just scale the variables independently.

So it said this: it said the Fourier transform of F of AX – all right, so you change the variables X by a matrix A, a nonsingular matrix A – is one over the determinate of A times the Fourier transform of F evaluated at A inverse transpose at the frequency variable C. Okay? It's a very interesting formula. We derived it last time, and it's complicated. It's more complicated than the one-dimensional stretch case, but it includes the one-dimensional stretch case, but what you don't see in one dimensions is this new phenomenon as I say that reciprocal somehow means inverse transpose. That's something new. That's something different on the scene. All right, so now I wanna show you how this comes in in these contexts. So first of all, I wanna talk about how we might generalize the Shah function to higher dimensions, so let me remind you of the Shah function in one dimension, and some of its main important properties. So I want to generalize the Shah distribution to the sum of deltas. Let me say evenly spaced deltas. So in one dimension we had the model case. That was a Shah function where the deltas are spaced by one, so you put a delta at each integer point. The model case is the basic Shah function where you put a delta function at each integer, zero, one, two, minus one, minus two, and so on. And you define the Shah as just the sum of these delta functions. Shah of X is the sum of K going from minus infinity to infinity at delta X minus K, so there's a delta function at each integer point.

And the remarkable property – and as we said when we were talking about this, this is the deepest property known about the integers. Its equivalent to this fact in number theory called the Poisson summation formula is the Fourier transform of the Shah function is itself. The Fourier transform of Shah is Shah. And that was fundamental in a lot of the work we did. It was fundamental also into the sampling theorem, and also into other applications. We actually looked at this in a slightly more general version, which I'll remind you of just now, in the context of crystallography, sort of as a way of motivating it, and I'm gonna talk exactly about that in just a second. So we generalize this slightly. We allow for spacing P for other spacing other than the integers, so spacing P. Doesn't matter what I call it. So instead of space to the integers, it's space – I put deltas at the points zero, P, 2P, and so on, and so on, minus P, minus 2P. So this time the Shah function is the same sort of definition but the spacing is different. So again it's the sum from minus infinity to infinity of delta X minus K times P. Here the fundamental result is that when you take the Fourier transform, the spacing becomes reciprocal. That is the Fourier transform of – let me use the subscript here P to indicate that that's what the spacing is. Sorry. The Fourier transform of the Shah function with spacing P is actually – there's two places where the reciprocal comes in. It gets scaled out in front by one over P, and then the spacing of the deltas in the frequency domain – the spacing of the deltas when you take the Fourier transform is spaced by one over P.

That's pretty cool. It's a very interesting result, and it's a very important example of this sort of reciprocity relationship between the two domains. And this is what I wanna generalize. All of these properties I wanna generalize. Now, I'm gonna stick in two dimensions, but really – and there's a change from one dimension to two dimensions, but there's very little change from two dimensions to higher dimensions. I can draw the pictures more easily in two dimensions, but really all I have to say will hold in higher dimensions, and in particular three dimensions, which is really the appropriate setting for studying crystals. But as I say just so I can draw the pictures a little more easily, I'm gonna write things and speak about things just in two dimensions. So what's the generalization of Shah let's say to 2-D. What should it be? Well, what I wanna do is –

there are two parts to the Shah function. I put a bunch of delta functions, but before I put the delta functions down, I have to have a set of evenly spaced points. That was the whole idea behind the Shah function, so you need – that's the first thing that you want. You want evenly spaced points, and you have to decide what that means, evenly spaced points in the plane, in R2. Now there are a lot of different possibilities here, but again we started with a Shah function with a model case where the deltas or the points were spaced one apart. So for my model of evenly spaced points in the plane, I'm gonna take all the points in two dimensions whose coordinates are integers. So I'll take a grid in the plane whose points have integer coordinates like so.

So that's gonna be my model for evenly spaced points in the plane. And of course, in the higher dimensions it would be a similar sort of thing. I would take points with integer coordinates in R3 or in higher dimensions. So a point here, a grid point has coordinates K1 and K2 where K1 and K2 are integers, all these points. Those points are said to form a lattice. That's not an unfamiliar term. So this forms what's called the integer lattice in the plane. It's denoted by Z2, sort of a bold-faced letter Z. I can't remember if I've used that notation before, Z for the integers, to stand for the integers. Z stands for the German word zahlen, which means number. It's a universally adopted term in – pretty much a universally adopted term in certainly mathematics and more and more so in engineering in application just to denote the integers. So Z2 meaning it's in two dimensions. So this is the set of all points, nothing but the set of all points and the lattice points in the plane with integer coordinates. And my Shah function is gonna be defined by putting a delta at each one of those points. Put a two-dimensional – put a spike at each one of those points. So define by Shah by putting a delta at each one of those points, each grid – I'll call it lattice points, okay? That is I'll define the Shah function for this integer lattice as exactly the analog of the one-dimensional case of X. It's gonna be the sum – let me write it like this: K in Z2 of delta X minus K. So again, what I'm trying to do here is I'm trying to make this look as much like the one-dimensional case as possible. So if I write the vector K, what I mean by that is it's a pair, K1, K2 where K1 and K2 are integers. So if I write it like that again it looks like the one-dimensional case. Here again K is just shorthand notation K1, K2, integers.

The basic phenomenon – again, generalizing the one-dimensional case is what happens to the Fourier transform of the Shah function for the integer lattice. And this is unfortunately something I don't feel like I really have the time to derive, and it just kills me, but I'll give you the result. So just as in the 1-D case, you have with the Fourier transform – and for the same reasons really, it's a straightforward – you set things up so it looks exactly the same as in the one-dimensional case. You have the Fourier transform, the two-dimensional Fourier transform of this lattice – the Shah function based on the integers is itself. The proof really again, the derivation of that is the same as in the one-dimensional case. It depends on – actually, it depends on higher dimensional Fourier series, but it's really very much the same. You can use the same words and really follow the same argument. The argument is very much the same, very much like the 1-D case. Now, things get more interesting when you ask yourself, "How do I change the spacing?" In the line, in one dimension, you only have one degree of freedom. The only way to change the spacing is to stretch or shrink in one direction. So how do I change the spacing for a 2-D lattice? What should we mean by that? We still want the points to be somehow evenly spaced, but not the same. Well again, in two dimensions, we have more degrees of freedom than in one dimension, but for our purposes, there's a very natural way of looking at this. In two dimensions, we obtain a different lattice. Informally, I'll call it an oblique lattice. I'll draw a picture in just a second. That is we change the spacing by applying a linear – a two by two matrix say A to the integer lattice.

That's what it is in words. Let me draw a picture so you see what I mean here because it's not hard to see what I mean. So here let me draw a picture of the integer lattice over here. So this is Z2. And as a matter of fact, let me take – let's consider this the origin, and let's take this to be the natural basis of R2, so the two natural basis vectors one zero and zero one. Now I take a linear transformation, a two by two matrix A nonsingular, so nothing's getting collapsed. Then they're gonna take those basis vectors E1 and E2 to some other vector, say V1 and V2. And they're gonna take – now all the points in the integer lattice are just integer combinations of these basis vectors. If I wanted to reach the Point 2 over here, then I just take two times E1. If I wanna reach the Point 2 up here, I take two times E2. If I wanna reach this point, I take E1 plus E2. Okay? One times E1 plus one times E2, and so on and so on. So all the points in the integer lattice are just integer combinations of the basis vectors E1 and E2. Under a matrix or linear transformation, they go over to integer linear combinations of V1 and V2, so what I have over here is a picture that looks like this where the lattice points over here correspond exactly to the lattice points over here, and so on and so on.

So the lattice is sheared if you wanna think of it that way. The square lattice of the integers is made into an oblique lattice by taking a matrix that takes the basis vectors over here, the natural basis vectors and some other pair of vectors. So A of Z2 – A applied to – I just mean by this applying A to all the different basis – all the different points in Z2 results in an oblique lattice L. Okay? And L is just the combination – the points in L are combinations of the basis vectors K1 times V1 plus K2 times V2 where K1 and K2 are integers. And here V1 say is A of E1 and V2 is A of E2. Stay with me. Now, one more geometric – actually, there are a couple of other things I have to introduce. It's very common – as a matter of fact, crystallographers do this – do we have anybody who has actually done any crystallography in here? By the way, I'm just curious. Then this is all new.

But if you do any kind of imaging, you're gonna see these terms and see these ideas, especially these days in molecular imaging and things like that where these things are coming in in really interesting ways. As a quick geometric description – so to jump ahead a little bit, and I'm gonna come back to this in a second, crystals come into this because you consider crystals to be modeled on lattices where there's an atom at each lattice point. And of course, the idea of considering oblique lattices is because some crystals are modeled on a rectangular lattice or a cubic lattice, but a lot of crystals aren't. There's sheer. They're oblique. There's a geometric – just to get an idea of somehow just to attach a number that describes a lattice in some loose but helpful sense, you often talk about the area of a lattice. So you often speak of the area of really what's called a fundamental cell, or fundamental parallelogram, fundamental cell in the lattice. And that's just the area of one of the parallelograms spanned by the basis vectors. So again, if here's the lattice, and say this is V1, V1 at V2, then the area of the lattice is the area of the – we're gonna come back – we're gonna need this concept in just a second. You talk about the area of the lattice as just the area of the parallelogram determined by the basis vectors, so the area of L is area of the parallelogram.

The reason why crystallographers think about that is because they think of that as the fundamental cell or fundamental building block for the rest of the crystal. That region in space is the fundamental building block for the rest of the crystal, and the question is how big is that. They talk about the volume of the fundamental cell. Everything in crystallography of course is in three dimensions. I'm only doing things in two dimensions, so I'm talking about the area in terms of volume, but that's what they do. They talk in those terms. So the area of the integer lattice is one because each one of these basis vectors is the length of one, and they're perpendicular. The area of that square is just one. Now I wanted to introduce that, but the real thing I want to get to of course is what happens if we consider the Shah function for an oblique lattice, and what is its Fourier transform. So again considering the Shah function for oblique lattice is like considering a model for a crystal where the underlying lattice structure or underlying crystal structure is on an oblique lattice instead of on a square lattice, so we can consider – let's call it the Shah function of lattice L. I'm struggling here with my script letters.

So let me just write it like this, it's the sum over all points in the lattice, so those are all the lattice points, the grid points – I put a delta at each one of those points, say delta of X minus P. You put a delta function at each lattice point P. Now the real question is what is the Fourier transform. What happens to the Fourier transform? What is the Fourier transform of the Shah function associated with the oblique lattice? A lot's going on here, right? You have to have the notion of a lattice. You have to have the notion of an oblique lattice. You have to have the notion of a Shah function on the lattice. And now you're asking about the Fourier transform. I mean what levels of complexity have been added to this simple conversation. And the reason why – and again, I'll come back to this in a second, but the reason why you wanna know about the Fourier transform is because in honest to God scientific experiments on crystallography, when you're shining X-rays through a crystal, you get a bunch of spots, and the pattern of the spots is determined by the Fourier transform of the crystal that's doing the diffracting.

So if the crystal that's doing the diffracting is modeled by the Shah function for a lattice, what you know about is you wanna know about the Fourier transform of that Shah function for the lattice because that's what you're seeing. Those are the spots that you're seeing. So now finally, here's where this reciprocal relationship comes in and it's just great. And again unfortunately, and I really do feel badly about this, I don't have the time to talk too much about the motivation for this, or even derive many of the formulas, so I just wanna show you in some sense what the punch line is without the whole set up, and it just kills me, but I've got to. So I'm gonna put two other lattices on that board, but here's the integer lattice Z2. I have two linear transformations. I have my linear transformation A that's going to go down to the oblique lattice L. That's the sort of the lattice that I'm starting with. Here's the oblique lattice L. And then the crystallographers

– and even mathematicians, too, but for completely different reasons as far as I can tell – define what they call the dual or reciprocal lattice, L star. So L is A of Z2. L is the transformation A. Any lattice can be realized as a linear transformation applied to the integer lattice. So L is A applied to Z2. Then the crystallographers the reciprocal or dual lattice to be – guess what – A inverse transpose applied to Z2. So they take A inverse transpose – [Crosstalk]

Now that's also another linear transformation. It goes into another lattice, and I don't have great confidence in actually how I draw it, but something like this say, L star. And they call that the reciprocal lattice. And in fact, as you can easily check, the area of L star – that is to say the area of one of these fundamental polygons, polyhedral is – so I certainly didn't draw it very well because it doesn't look quite so easy – is the reciprocal of the area of the other lattice. Now this is one definition of it, and this is the cleanest definition. It's completely unmotivated. I'm sorry. There are different ways of motivating it. There's one discussion in the notes. That is it's a completely unmotivated definition. Why the hell would you define a dual lattice or reciprocal lattice this way? You just have to take my word for it or look around for other motivations for it, other definitions for it. In fact, people who work in materials and people who work in crystallography actually have a geometric construction of dual lattice. Given a lattice L, they have a way of constructing geometrically the dual lattice, like you know a ruler and compass construction. So it's a time honored important construction in materials to pass from the lattice to the reciprocal lattice.

Now, why? What is the big deal? Because the fundamental on the Shah function is that the Fourier transform for the Shah function for a lattice is the Shah function for the dual lattice except scaled by the area, so that's a fundamental fact. The Fourier transform – I've got too many Fs and Ls and everything else in here. The Fourier transform of the Shah function of a lattice L is – it's a beautiful formula – it's one over the area of L times the Shah function of the dual lattice. Now again in higher dimensions which is where the real applications of crystallography go, you'd have the same result except you'd have volume here instead of area, but it's the same kind of result, and the definition of a dual lattice is the same, except instead of a transformation in R2, instead of everything happening in R2 here, everything would be happening in R3. You'd have the lattice of integer points in R3. You'd have a three by three matrix going over to an oblique lattice here in R3. You'd have the inverse transpose going into another oblique lattice of R3 and so on. And you'd have the same result. This is exactly – if you believe it – analogous to the one-dimensional formula. This is the analog of the one-dimensional formula. This is the analog to the Fourier transform of the one-dimensional Shah function of with spacing P is one over P times the Shah function with spacing one over P.

There reciprocal just means one over. But see, the thing is you miss something. If you just look at that, you either miss something in the one-dimensional case, or something new and deeper is revealed in the higher dimensional case because in the one-dimensional case, reciprocal means reciprocal. It just means one over. In the higher dimensional case, reciprocal means inverse transpose. That's the change in point of view. That's sort of the change in intuition. It's very important. That's something you don't see in the one-dimensional case. That's a difference between the one-dimensional case and the two-dimensional case in how to understand the notion of reciprocity, to understand what reciprocal means. It's a mathematical fact, and not only a mathematical fact, a fact of nature that in higher dimensions reciprocal often means – can be interpreted as involves, ergo it [inaudible] reciprocal transpose. Now so what about crystals in all of this? I made allusions to this a lot, so let me just say quickly what the situation is for crystals and why this is so important or where this comes up. So again for crystals – so again when we did this in the one-dimensional case, the idea is you study a crystal by studying the electron density distribution of a crystal. What is the electron density distribution of a crystal? You see, you conjecture, you measure how the electrons are distributed about an atom, and then you periodize that. Crystals have a periodic structure, so row of X in two dimensions now is the electron density for an atom in a crystal. The electron density distribution for the whole crystal is a periodized version of that. For the crystal as a whole, you take a periodized version. That's the whole point about crystals is they have periodic structure. How do you take a periodized version? You convolve with the appropriate Shah function.

So if this is the crystal whose atoms are what you model as a bunch of atoms at lattice points as L, then the density for the lattice, the density for the crystal is the density for a single atom convolved with the Shah function. That's sum over all the points in the lattice of row of X minus P, same as in the one-dimensional case. So if there's a little – if the density looks like that there, then that pattern's just repeated for the whole lattice, and again I can't draw this too well, but you get the picture. This is now periodic with respect to the lattice. This is also a different phenomenon. It has two periods. It's periodic in that direction, and it's periodic in that direction. It's periodic on the whole lattice. The pattern repeats on the whole lattice. Now X-ray crystal experiments, X-ray diffraction experiments measure the Fourier transform. That is an X-ray diffraction experiment produces the magnitude of the Fourier transform, but just basically think of it as the Fourier transform – produces the Fourier transform of row of the lattice. That's what it does. You see a bunch of spots. What you're seeing is the Fourier transform of the lattice, and what is that? Well, that's the Fourier transform of the convolution, so that's the product of the Fourier transform of row with the Fourier transform of the Shah. This is the Fourier transform of row times the Fourier transform of the Shah function of the lattice.

But we know what that is. It's the dual lattice. That is this is the Fourier transform of row times one over the area of L times the Shah function of the dual lattice. And if I take the product of the function times a bunch of delta functions, what do I get? I get sum over the points – let me call it P star in the dual lattice. They're just delta functions at the lattice points of the dual lattice of the Fourier transform of row at – I don't know, S times delta of S minus P star. So the Fourier transform of the density function times the sum of deltas, and remember what happens when you take a function times delta? It just pulls out the value – sorry, P star. So once again, you do your experiment, and unless you know the math, you're not gonna be able to draw the right conclusion. You might think that you do – you pass a bunch of X-rays through the crystal, and the spacing of the lattice points that you're seeing should be proportional to the spacing of the lattice points in the crystal, but it’s not. Nature for whatever mysterious reason is taking a Fourier transform. X-ray crystallography, X-ray diffraction takes a Fourier transform for you physically. And what you’re seeing is you’re see the points at – you’re seeing the little spots that appear on your X-ray film at points on the dual lattice. And unless you know the math, unless you know that in higher dimensions reciprocal means inverse transpose and so on, you can kiss your Nobel prize goodbye.

But if you sort of follow your pencil through this, and see how the math plays out here, you can draw conclusions about the crystal by knowing this. I think it’s really cool. I think it’s a really interesting application and physical manifestation of this fact that in higher dimensions inverse transpose means reciprocal. So I don’t have time to derive this formula, and I’m sorry for that. The derivation of this formula – and that shouldn’t surprise you given how all these terms are coming – involves exactly the generalized stretch theorem. The reason why inverse transpose comes in here, and the reason why the dual lattice comes in is because in deriving this formula, it uses exactly the generalized stretch theorem. That’s how it – it comes into it in deriving that formula. Isn’t that nice? So I say I wish we had one additional day because I would like to go into just a little bit more detail about this, but we don’t. We don’t because I spent all that time on the damn fast Fourier transform. Everybody wanted to see that damn fast Fourier transform. So full stop, speaking about things I can’t spend enough time on, there’s one more topic I won’t be able to spend enough time on. The final topic in the class is application of the two-dimensional Fourier transform to the problems of medical imaging, and particularly to the problems of tomography. So we’ll leave this fascinating subject behind, and pass to another fascinating subject. I’ll start off today, and then we’ll finish it up next time.

So the final application is – the final topic is the application of the two-dimensional Fourier transform to medical imaging, and in particular the problem of tomography. I will tell you what the problem of tomography is in just a second – the fundamental problem of tomography. Now let me say at once that what I’m gonna be looking at is really in the realm of CAT scan, of computer tomography, of tomography not MRI. But as it turns out this is actually – this is not MRI. As they say in the biz, the modalities of imaging for the two kinds of – two approaches are different. However, it’s also true for whatever reason – well, for a reason that we can’t really get into that the final formula that I’m gonna get to solve the problem of tomography is the same as the formula you get when you’re using an MR system to do the imaging. So what we’re doing here, although it’s in the realm of tomography, CAT scans and things like that, actually the final formulas that you get and a lot of the considerations that we’re gonna talk about actually apply in the MR case, magnetic resonance case, NMR case, like nuclear magnetic resonance, but again I cannot get into it. Here’s the basic setup. You have a 2-D region, a two-dimensional region, think of it as a slice of your body filled with goop, blood, bones, organs, you know. There you are, and there’s all that goop inside you. And that goop inside you is described by a function of variable density. You can imagine describing the density of the goop. So describe the goop by a function mu of X1 X2. So this is taking place in the X1 X2 plane, and mu gives the density of the goop at each point.

So if you know mu, you know you, so to speak that is to say you wanna get you by taking measurements. You want to recover mu X1 X2 by X-ray measurements or let me just say by whatever. That's what you want. You want to reconstruct the function mu. Now here's how it's done in tomography. The approach to tomography is the following. Tomography means – if I get this right. I forgot to look it up to get the precise – I think it means section, so the idea is you are taking sections, and in particular actually what I'm gonna do is I'm gonna take a one-dimensional section. That is I'm gonna pass a line through this region. So the approach via tomography – a line meaning I'm gonna pass an X-ray. The approach via tomography is you shoot an X-ray through the region along a line and it comes out the other end. It gets diffracted. It get attenuated by all this goop in the middle. So here's the X-ray, and there's all this stuff in here, and you measure what happens to it when it comes out. So you know how – you know what it was going in, and you measure what it is coming out. Say it goes this way, so you know the intensity of the X-ray going in, and then it gets attenuated, so you measure the intensity – let's call this I not, the initial intensity. You measure the intensity going out on the other end. Let's call that I.

Now then it is not hard to show as a reasonable approximation – and again, this is derived in the notes. You can show that there's a relatively simple – well, relatively simple. We'll see. There's a relationship between the intensity going out and the intensity coming in. This is I not going in, I coming out, and it's attenuated – it drops exponentially, so it's given by intensity coming out is the intensity going in, and then I say it drops exponentially. How? It drops exponentially according to the integral of mu along the line, sort of like the average density. Mu is a variable density, but the total drop in intensity is given by E to the minus integral – so integral of L over mu – I'm using a shorthand notation here – is the line integral of mu along L. Again, you can sort of think of that as the average density or whatever. There are different ways of making that argument, and again I don't wanna take the time now to derive that formula for you, but it's actually not that hard to derive, and it's not even that hard to believe.

Now again, you know I not. You measure I. So all those numbers are known. What you don't know is you don't know mu, and in particular you don't know the integral of mu along a line. What you do is you make lots of measurements. You send X-rays through along all different lines, or along whole families of lines. You send X-rays through along many lines, this way, that way. Usually you think of a parallel line, but then you change the angle. You send them in this way. Don't think of this as a lattice. I'm just changing my X-rays here. You send a bunch of lines in. Then you know, and at every time you know the intensity going in, you measure the intensity coming in, so that means you know the integral along all those different lines of mu. Once again, you write this down. I equals I not E to the minus integral of mu over the line L for all different L. You know this. You measure this. So then you know all the numbers integral over L of mu. When I say all the numbers, you know them along all these different lines. It's because you made those measurements.

And the fundamental problem of tomography as it is often stated is can you recover mu by knowing all these line integrals. That's the problem. So again, I wanna make sure you understand here at least what you know and what you're trying to figure out. You know the value of these integrals because you know this formula, and you know the intensity going in, and you measure the intensity coming out, so that tells you what this. You just solve for it in terms of log. So the question is – the fundamental problem of tomography is can you recover mu by knowing the integral of mu along L for all L, for all lines L. That's the question. And the answer is affirmative. The answer is in the affirmative. The answer is yes, and it involves the two-dimensional Fourier transform. That's how it comes into the picture. I'm not gonna get to that today, but I'll tell you how we're gonna approach it actually. So the answer is yes. It's not obvious, and it's certainly not obvious that the Fourier transform's gonna come into this in any sort of way, but here's the approach. This is the approach that we're gonna take, and here's how it works. And as I say, it's not obvious. It was a brilliant idea. You consider these numbers as defining a transform of mu. What I mean by that is you start with a line – what's that a function of? It depends on the line. The number depends on the line.

So starting with a line L, you compute the number, the integral of mu along L. L goes to – that's a line – goes to the integral over L of mu. That's a number. When I say transform is you wanna consider this a transform of mu evaluated on the line L. Now that's sort of mind expanding for you, but that's the way you look at it. That is write this as R of mu, the transform of mu at a particular line, so again that is by definition that integral. That's the integral along L of mu. That's called the Radon transform of mu. We're almost out of [inaudible]. It's called the Radon transform of mu. The fundamental problem of tomography can be stated as saying can you invert the Radon transform. R of mu gives you something [inaudible] L. The question is can you find mu given all the values of R of mu. Can we invert? Is there an inverse Radon transform to find mu? And the answer is yes, and the answer is in terms of the two-dimensional Fourier transform. It's amazing. It's just amazing, but that's what's gonna happen, so that's why the two-dimensional Fourier transform, and that's why Fourier transform techniques come into medical imaging because of exactly this connection.

So we will finish up with that next time, and me, I'm gonna go take a pill and collapse. Thank you all. I'll see you all on Friday.

[End of Audio]

Duration: 51 minutes